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PRACTICAL CALCULATION OF TRANSMISSION LINES

PRACTICAL CALCULATION
OF
TRANSMISSION LINES

FOR DISTRIBUTION OF DIRECT AND ALTERNATING CURRENTS BY MEANS OF OVERHEAD, UNDERGROUND, AND INTERIOR WIRES FOR PURPOSES OF LIGHT, POWER, AND TRACTION

BY

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PREFACE.

THIS little book is offered to the engineering profession with the hope that it may be of practical help in the rapid and accurate calculation of transmission lines. Its existence is the outcome of the belief that this field is in part barren. Its chief mission is to substitute a direct solution for the trial method which was formerly a necessary evil. The arrangement of the formulas, tables and text has been dictated solely by the needs of the rapid worker.

All sections except the last include the important effects of temperature and specific conductivity. The section relating to direct-current railways is novel in the form of its tables, and the methods outlined in it have been found rapid and comprehensive. The alternating-current division presents a new and original method for the solution of these problems. It is the only method known to the author which determines the size of wire directly from the volt loss in the line, and it also possesses unique features of scope, accuracy and simplicity. The chapter on single-phase railways is in accord with most of the consistent data published on the subject, although further accurate investigations of installed lines may modify to some extent the present accepted values.

The author desires to call particular attention to the fallacies of some familiar methods of calculating alternating-current transmission lines which heretofore have been in common use. It will be evident from Tables 11, 28, and 36 that their results are wholly erroneous under certain practical conditions, and indicate wires which may be entirely at variance with the specified requirements.

The scope of this book has been confined to methods of calculation. Hence, the most desirable limits of line losses are not discussed, but the tables are sufficiently extended to cover all cases likely to arise in practice. There is no discussion of the characteristics of alternating-current transmission lines, although the tables of wire factors render apparent many of their important features. Furthermore, the book does not include either the determination of

the size of conductors for conditions of maximum economy or the consideration of alternating-current circuits in series.

The preparation of the tables has involved thousands of calculations, but thorough checks by the methods of differences and curve plotting have probably eliminated almost all errors of material influence. However, some discrepancies may have crept in, and the author would be glad to learn of them.

L. W. R.

NEW YORK CITY, *December, 1908.*

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PART I.

DIRECT-CURRENT DISTRIBUTION BY MEANS OF OVERHEAD, UNDERGROUND AND INTERIOR WIRES FOR PURPOSES OF LIGHT, POWER AND TRACTION

DIRECT-CURRENT DISTRIBUTION

CHAPTER I.

DIRECT-CURRENT DISTRIBUTION FOR LIGHT AND POWER.

1. **Introduction.** — Problems in direct-current transmission and distribution are relatively simple. The same formula covers all conditions of installation and operation whether by overhead, underground or interior wires. The formulas give accurate results, and all items of influence are easily included. The tables are concise but comprehensive and will cover almost all the usual and unusual requirements of varied practice.

2. **Properties of Conductors.** — The resistance of stranded and solid conductors of the same cross section and length is practically the same. Table 3, page 8, gives the properties of wires at 20° cent. or 68° fahr. for copper of 100 per cent and aluminum of 62 per cent conductivity in Matthiessen's standard scale. The resistance at any other temperature and conductivity may be found for copper or aluminum from formulas (1) and (2).

$$\text{Ohms resistance per 1000 feet} = \frac{10{,}350 \times T}{S} \quad . \quad . \quad . \quad (1)$$

$$\text{Ohms resistance per mile} = \frac{54{,}700 \times T}{S} \quad . \quad . \quad (2)$$

S = Cross section of metal in circular mils.

T = Temperature factor. Table 1, page 6.

Thus, at 40° cent. the resistance per mile of No. 1 copper wire of 98 per cent conductivity is

$$\frac{54{,}700 \times 1.099}{83{,}690} = 0.718 \text{ ohm.}$$

Table 1.—Values of T for Copper and Aluminum.

Temperature in degrees.		Copper.					Aluminum
		Conductivity in Matthiessen's standard scale.					
Cent.	Fahr.	1.00	0.99	0.98	0.97	0.96	0.62
0	32	0.926	0.936	0.946	0.955	0.965	1.495
10	50	0.963	0.973	0.983	0.993	1.003	1.553
20	68	1.000	1.010	1.020	1.031	1.042	1.613
30	86	1.038	1.048	1.059	1.069	1.081	1.674
40	104	1.077	1.088	1.099	1.110	1.122	1.738
50	122	1.117	1.128	1.140	1.150	1.163	1.802

For copper or aluminum of other conductivity, divide value under 1.00 by given conductivity.

The weight of a unit length of copper or aluminum wire may be calculated from the following:

$$\text{Pounds per unit length} = FS. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

F = Weight factor. Table 2, page 6.
S = Cross section of metal in circular mils.

Table 2.—Values of F for Copper and Aluminum.

Unit Length.	Copper Wires.		Aluminum Wires	
	Solid.	Stranded.	Solid.	Stranded.
1000 feet....	0.00303	0.00305	0.00091	0.00092
Mile....	0.0160	0.0161	0.00482	0.00486

3. **Current-carrying Capacity.** — Table 4, page 8, gives the current-carrying capacity of wires under various conditions. Overhead wires larger than No. 0 have been considered stranded, while those smaller than No. 0 have been taken solid. The values for interior wires have been taken from the National Electric Code. The results for bare and insulated overhead wires are calculated from formula (4) for copper of 100 per cent conductivity and a temperature rise of 40° cent. or 72° fahr. The

Table 3.—Properties of Copper and Aluminum at 20° Cent. or 68° Fahr. Conductivity in Matthiessen's Standard Scale; Copper 100 Per Cent, Aluminum 62 Per Cent.

B.& S. or A.W.G.	Area, Cir. Mils.	Diameter, Inches.	Copper.				Aluminum.			
			Ohms Resistance.		Pounds Weight.		Ohms Resistance.		Pounds Weight.	
			1000 Ft.	Mile.	1000 Ft.	Mile.	1000 Ft.	Mile.	1000 Ft.	Mile.
			Bare Stranded Wires.							
.....	1,000,000	1.152	0.0104	0.0547	3050	16100	0.0167	0.0882	920	4860
.....	800,000	1.035	0.0129	0.0683	2440	12880	0.0209	0.110	736	3890
.....	700,000	0.963	0.0148	0.0781	2140	11270	0.0239	0.126	644	3400
.....	600,000	0.891	0.0173	0.0911	1830	9660	0.0278	0.147	552	2920
.....	500,000	0.819	0.0207	0.109	1530	8050	0.0334	0.176	460	2430
.....	400,000	0.728	0.0259	0.137	1220	6440	0.0417	0.220	368	1940
.....	300,000	0.630	0.0345	0.182	915	4830	0.0557	0.294	276	1460
.....	250,000	0.590	0.0414	0.219	762	4030	0.0668	0.353	230	1220
0000	211,600	0.530	0.0489	0.258	645	3410	0.0790	0.417	195	1030
000	167,800	0.470	0.0617	0.326	513	2710	0.0997	0.526	154	816
00	133,100	0.420	0.0778	0.411	406	2140	0.126	0.664	122	647
0	105,500	0.375	0.0981	0.518	322	1700	0.158	0.837	97.1	513
1	83,690	0.330	0.124	0.653	255	1350	0.200	1.06	77.0	407
2	66,370	0.291	0.156	0.824	203	1070	0.252	1.33	61.0	323
3	52,630	0.261	0.197	1.04	160	845	0.318	1.68	48.5	256
4	41,740	0.231	0.248	1.31	127	671	0.401	2.12	38.5	203
			Bare Solid Wires.							
0000	211,600	0.460	0.0489	0.258	641	3380	0.0790	0.417	193	1020
000	167,800	0.410	0.0617	0.326	508	2680	0.0997	0.526	153	809
00	133,100	0.365	0.0778	0.411	403	2130	0.126	0.664	121	640
0	105,500	0.325	0.0981	0.518	320	1690	0.158	0.837	96.1	508
1	83,690	0.289	0.124	0.653	253	1340	0.200	1.06	76.3	403
2	66,370	0.258	0.156	0.824	201	1060	0.252	1.33	60.4	320
3	52,630	0.229	0.197	1.04	159	841	0.318	1.68	48.0	253
4	41,740	0.204	0.248	1.31	126	667	0.401	2.12	38.1	201
5	33,100	0.182	0.313	1.65	100	529	0.505	2.67	30.2	159
6	26,250	0.162	0.394	2.08	79.5	420	0.637	3.37	23.9	127
8	16,510	0.129	0.627	3.31	50.0	264	1.01	5.35	15.1	79.5
10	10,380	0.102	0.997	5.27	31.4	166	1.61	8.51	9.5	50.0
12	6,530	0.0808	1.59	8.37	19.8	104	2.56	13.5	6.0	31.8
14	4,107	0.0641	2.52	13.3	12.4	65.6	4.07	21.5	3.7	19.8
16	2,583	0.0508	4.01	21.2	7.8	41.3	6.48	34.2	2.4	12.4
18	1,624	0.0403	6.37	33.7	4.9	26.0	10.3	54.4	1.5	7.8

Table 4. — Ampere-Feet per Volt Drop and Current-Carrying Capacity.

Cir. Mils or A.W.G.	Ampere-Feet per Volt Drop at 20° Cent.*		Current-Carrying Capacity in Amperes per Wire.							
			Interior Wires. Nat'l Elec. Code.†		Overhead Bare Wires.		Overhead Wires, Rubber Insulation.		Underground Copper Cables.	
	Copper.	Aluminum.	Rubber.	W'th'r-proof.	Copper.	Aluminum.	Copper.	Aluminum.	1-Condr	3-Condr
1000000	48,100	29,900	650	1000	2460	1950	1090	870	690	.....
800000	38,700	23,900	550	840	2080	1670	930	730	610	.....
700000	33,800	20,900	500	760	1880	1500	840	660	560	.....
600000	28,900	18,000	450	680	1670	1330	750	590	500	.....
500000	24,100	15,000	390	590	1480	1170	660	520	450	.....
400000	19,300	12,000	330	500	1240	980	550	440	390	.....
300000	14,500	9,000	270	400	990	790	440	350	320	250
250000	12,100	7490	240	350	900	720	400	320	290	220
0000	10,200	6330	210	312	760	610	340	270	260	190
000	8110	5020	177	262	630	500	280	230	220	170
00	6430	3970	150	220	540	430	240	190	190	150
0	5100	3170	127	185	460	370	200	160	170	130
1	4030	2500	107	156	350	280	150	120	150	110
2	3210	1980	90	130	300	250	120	100	120	94
3	2540	1570	76	110	250	210	100	84	110	81
4	2010	1250	65	92	210	170	88	69	91	68
5	1600	990	54	77	170	140	74	59	76	57
6	1270	785	46	65	150	120	63	50	64	48
8	798	495	33	46	100	82	44	35	45	.....
10	502	311	24	32	.....	.....	.....	.....	.....	.....
12	314	195	17	23	.....	.....	.....	.....	.....	.....
14	198	123	12	16	.....	.....	.....	.....	.....	.....
16	125	77	6	8	.....	.....	.....	.....	.....	.....
18	78	49	3	5	.....	.....	.....	.....	.....	.....

* Product of feet of one wire and amperes for total drop of one volt in both wires.
† These values are for copper wires; for aluminum wires, multiply by 0.84.

Table 5. — Values of *H* for Copper or Aluminum.

Type of Installation.	Solid Wires.	Stranded Wires.
Overhead wires, bare	389	342
Interior wires, exposed and bare.....	205	190
Overhead wires, rubber insulation.....	165	152

current per wire for lead-covered underground cables is based on tests for a temperature rise from 70° fahr. initial to 150° fahr. final.* Formula (4) may be solved for C to find the temperature rise under given conditions.

$$\text{Amperes per wire} = H\sqrt{\frac{Cd^3}{T}} \quad . \quad . \quad . \quad . \quad . \quad (4)\dagger$$

$d =$ Diameter of wire in inches.
$C =$ Temperature rise in degrees centigrade.
$H =$ Heat factor. Table 5, page 8.
$T =$ Temperature factor at final temperature. Table 1, page 6.

The size of wires is sometimes determined by their current-carrying capacity, especially in interior work where the runs are short. For longer lines it is usually advisable to calculate the loss and then note that the wire has sufficient carrying capacity, while for shorter stretches it is often better to select a wire of proper current capacity and then find by calculation whether the loss is within the specified limit.

4. **Parallel Resistance of Wires.** — The parallel resistance at any temperature for a number of wires of any conductivity is given by the following formulas:

$$\left.\begin{aligned} \text{Ohms resistance per 1000 feet} &= \frac{10{,}350}{\frac{S_1}{T_1} + \frac{S_2}{T_2} + \cdots} \quad . \quad . \quad . \\ \text{Ohms resistance per mile} &= \frac{54{,}700}{\frac{S_1}{T_1} + \frac{S_2}{T_2} + \cdots} \quad . \quad . \quad . \end{aligned}\right\} \quad (5)$$

Where all the wires have the same temperature and conductivity, formulas (5) reduce to those below.

$$\left.\begin{aligned} \text{Ohms resistance per 1000 feet} &= \frac{10{,}350 \times T}{S_1 + S_2 + \cdots} \quad . \quad . \\ \text{Ohms resistance per mile} &= \frac{54{,}700 \times T}{S_1 + S_2 + \cdots} \quad . \quad . \end{aligned}\right\} \quad (6)$$

In formulas (5) and (6)

S_1, $S_2 =$ Circular mils in respective wires.

T, T_1, $T_2 =$ Temperature factors of respective wires. Table 1, page 6.

* Standard Underground Cable Co.'s Handbook No. XVII, page 192.

† Based on formulas in Foster's "Electrical Engineers' Pocketbook," 1908, page 208.

Thus, the resistance per mile at 30° cent. of one No. 00 trolley wire of 97 per cent conductivity in parallel with one 1,000,000-cir. mil aluminum feeder of 62 per cent conductivity is

$$\frac{54{,}700}{\frac{133{,}100}{1.069} + \frac{1{,}000{,}000}{1.674}} = 0.0758 \text{ ohm.}$$

Similarly, the parallel resistance per 1000 feet at 40° cent. of one No. 0 and one No. 4 copper wires of 97 per cent conductivity is

$$\frac{10{,}350 \times 1.110}{105{,}500 + 41{,}740} = 0.0780 \text{ ohm.}$$

5. **Given Items.** — The problem may require the determination of the size of wire from the volt loss, or the volt loss from the given size of wire. In either case the other required items are conductor metal and temperature, current and distance of transmission. If power in watts is specified, then the voltage at load or source must also be given.

The term "source" is used in this book to designate the point from which the circuit, or the part of circuit under consideration, starts. Thus in direct-current distribution it may signify the generator, rotary converter, storage battery, connection to main, to feeder or to sub-feeder, or simply a certain point of the circuit.

6. **Formulas.** — Formulas for the complete solution of direct-current problems are given on page 14, and the method of procedure will be clear from the arrangement of the table. The size of wire may be determined from the volt drop, or from the per cent volt drop, which in direct-current systems is always equal to the per cent power loss. The value of a is found in Table 6, page 15, and the size of each wire is noted from Table 3, page 7, corresponding to the required circular mils.

The per cent volt drop is expressed in terms of power as well as current, so that problems involving voltage at source and watts at load may be solved without preliminary approximation. It should be carefully noted that per cent loss, V or V_0, is expressed as a whole number, and that the length of transmission, l, is the distance from source to load, which is the same as the length of one wire. In formula (14), r is the resistance per foot of one wire, equal to the values in Table 3, columns 4 or 8, divided by 1000.

7. **Ampere-Feet.** — The size of wires may be readily determined from the ampere-feet per volt drop as given by formula (7).

The wire having a corresponding value is then noted from Table 4, page 8.

$$\text{Ampere-feet per volt drop} = \frac{Il}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

I = Amperes.
l = Distance from source to load, in feet.
v = Drop in volts.

The values in Table 4 are for copper and aluminum of 100 per cent and 62 per cent conductivity, respectively, at 20° cent. For other conductivities or temperatures, formula (8), page 14, is more convenient.

In case of distributed loads, as in interior lighting work, Il is the sum of the products given by each load, I, multiplied by its respective distance, l. Thus if 6 amperes are to be transmitted 25 ft., 3 amperes 50 ft., and 2 amperes 100 ft.,

$$Il = 6 \times 25 + 3 \times 50 + 2 \times 100 = 500 \text{ ampere-feet.}$$

8. Examples. — Examples in practice may take innumerable forms, but the method of procedure in any case will be clear from the table of formulas on page 14. The following problems are typical and serve to illustrate the simplicity of the calculations for direct-current distribution.

Example 1. — A copper circuit of 97 per cent conductivity is to deliver 200 amperes for a distance of 1000 feet with a loss of 10 volts at 40° cent.

GIVEN ITEMS.

I = 200 amperes; v = 10 volts; l = 1000 feet.
From Table 6, page 15, a = 23.0.

REQUIRED ITEMS.

Size of each wire. — From (8),

$$S = \frac{23 \times 200 \times 1000}{10} = 460{,}000 \text{ cir. mils.}$$

Use 500,000 cir. mil wires.

Volt drop. — From (9),

$$v = \frac{23 \times 200 \times 1000}{500{,}000} = 9.2 \text{ volts.}$$

Watt loss. — From Tables 1 and 3,

$$T = 1.110, \text{ and } r = 0.0000207 \text{ ohm.}$$

From (14),

$$p = 2 \times 1.11 \times 0.0000207 \times (200)^2 \times 1000 = 1840 \text{ watts.}$$

By assuming the voltage at the load at any value, say 100, the per cent drop and per cent power loss are found to be the same, 9.2 per cent.

Example 2. — A motor is to take 25 kw. at a distance of 240 feet with a loss of 5 per cent of the 110 volts generated, while the circuit is to consist of copper wires of 98 per cent conductivity with a temperature of 30° cent.

GIVEN ITEMS.

$w = 25{,}000$ watts; $e_0 = 110$ volts; $V_0 = 5$ per cent; $l = 240$ feet.

From (19), $v = 0.05 \times 110 = 5.5$ volts.

From (17), $$I = \frac{25{,}000}{110\,(1 - 0.05)} = 239 \text{ amperes.}$$

From Table 6, page 15, $a = 21.9$.

REQUIRED ITEMS.

Size of each wire. — From (8),

$$S = \frac{21.9 \times 239 \times 240}{5.5} = 228{,}000 \text{ cir. mils.}$$

Use No. 0000 wires, for which $S = 211{,}600$ cir. mils.

Per cent volt drop. — From (11),

$$V_0''' = \frac{21.9 \times 25{,}000 \times 240}{0.01 \times 211{,}600 \times (110)^2} = 5.1.$$

From Table 31, page 72, $V_0 = 5.4$ per cent.

Volt drop. — From (19), $v = 0.054 \times 110 = 5.9$ volts.

Volts at load. — From (13), $e = 110 - 5.9 = 104.1$ volts.

Watt loss. — From (14), $$p = \frac{0.054 \times 25{,}000}{1 - 0.054} = 1430 \text{ watts.}$$

Watts at generator. — From (15),

$$w_0 = 25{,}000 + 1430 = 26{,}430 \text{ watts.}$$

From Table 4, page 8, it is seen that the wire should have weatherproof insulation or else be increased to 250,000 cir. mils.

Example 3. — The load on a feeder is to consist of 20 amperes at 50 feet, 25 amperes at 100 feet, and 40 amperes at 150 feet, from the source. Calculate the required size of a uniform circuit of 100 per cent conductivity for a total loss of 2 volts at 20° cent.

From (7), Ampere-feet per volt drop

$$= \frac{20 \times 50 + 25 \times 100 \times 40 \times 150}{2} = 4750.$$

From Table 4, page 8, the required size of each wire is No. 0 for copper and No. 000 for aluminum.

Example 4. — Copper mains of 98 per cent conductivity are to deliver 500 amperes to a point 550 feet from a rotary converter with a loss of 3 per cent of the voltage at load. Calculate the size of wire of 98 per cent conductivity and at 50° cent. if 220 volts are generated.

GIVEN ITEMS.

$I = 500$ amperes; $e_0 = 220$ volts; $V = 3$ per cent; $l = 550$ feet.

From (18), $$v = \frac{0.03 \times 220}{1 + 0.03} = 6.4 \text{ volts.}$$

From Table 6, page 15, $a = 23.6$.

REQUIRED ITEMS.

Size of each wire. — From (8),

$$S = \frac{23.6 \times 500 \times 550}{6.4} = 1{,}010{,}000 \text{ cir. mils.}$$

Use 1,000,000 cir. mils.

Volt drop. — From (9), $v = \dfrac{23.6 \times 500 \times 550}{1{,}000{,}000} = 6.5$ volts.

Per cent volt drop. — From (19), $V_0 = \dfrac{6.5}{2.20} = 2.95$ per cent.

Volts at load. — From (13), $e = 220 - 6.5 = 213.5$ volts.

Watts at load. — From (16), $w = 213.5 \times 500 = 106{,}750$ watts.

Example 5. — Find the combined resistance of 1500 feet of 500,000 cir. mils of 98 per cent copper wire at 20° cent. in parallel with the same length of 1,000,000 cir. mils of aluminum wire of 62 per cent conductivity at 30° cent.

From (5) and Table 1,

$$\text{Resistance} = \frac{10{,}350 \times 1.500}{\dfrac{500{,}000}{1.02} + \dfrac{1{,}000{,}000}{1.674}} = 0.0143 \text{ ohm.}$$

Formulas For Direct-Current Wiring.

Required Items.	For Voltage Given at Load.	For Voltage Given at Source.
Circular mils in each wire.	a from Table 6, page 15.	
	$S = \frac{aIl}{v}$ . . . . . (8)	
	Then find wire from Table 3, page 7.	
Volt drop.	$v = \frac{aIl}{S}$ . . . . . (9)	
Per cent volt drop or Per cent power loss.	$V = \frac{aIl}{0.01\,Se} = \frac{v}{0.01\,e}$. (10)	$V_0''' = \frac{awl}{0.01\,Se_0^2}$ . . (11)
		V_0 from Table 31, page 72.
Volts at source or load.	$e_0 = e + v = e\,(1 + 0.01\,V)$ (12)	$e = e_0 - v = e_0(1 - 0.01\,V_0)$ (13)
Watt loss.	T from Table 1, page 6. r from Table 3, page 7.	
	$p = 2\,TrI^2l = 0.01\,Vw = \frac{0.01\,V_0w}{1 - 0.01\,V_0}$ . . . . (14)	
Watts at source.	$w_0 = w + p = w\,(1 + 0.01\,V) = \frac{w}{1 - 0.01\,V_0}$. (15)	
Amperes.	$I = \frac{w}{e}$ . . (16)	$I = \frac{w}{e_0\,(1 - 0.01\,V_0)}$. (17)

When given V, find v from $v = 0.01\,Ve = 0.01\,Ve_0\,/\,(1 + 0.01\,V)$. . . (18)

When given V_0, find v from $v = 0.01\,V_0e_0 = 0.01\,V_0e/\,(1 - 0.01\,V_0)$ . . (19)

When given w, find I from (16) or (17) above.

NOTATION.

a = Resistance factor. Table 6, page 15.
e = Volts at load.
e_0 = Volts at source.
I = Amperes from source.
l = Distance from source to load, in feet.
p = Total power loss in watts.
r = Resistance per foot of one wire, in ohms. Table 3, page 7.
S = Circular mils in each wire.
T = Temperature factor. Table 1, page 6.

V = { Volt drop in per cent of volts at load. / Power loss in per cent of power at load. }

V_0 = { Volt drop in per cent of volts at source. / Power loss in per cent of power at source. }

V_0''' = Volt loss factor. Table 31, page 72.
v = Volt drop.
w = Watts at load.
w_0 = Watts at source.

Table 6. — Values of *a* For Copper and Aluminum.

Temperature in degrees.		Copper.					Aluminum.
		Conductivity in Matthiessen's standard scale.					
Cent.	Fahr.	1.00	0.99	0.98	0.97	0.96	0.62
0	32	19.2	19.4	19.6	19.8	20.0	30.9
10	50	19.9	20.1	20.3	20.5	20.8	32.2
20	68	20.7	20.9	21.1	21.3	21.6	33.4
30	86	21.5	21.7	21.9	22.2	22.4	34.7
40	104	22.3	22.5	22.7	23.0	23.2	36.0
50	122	23.1	23.3	23.6	23.8	24.1	37.3

For copper or aluminum of other conductivity, divide value under 1.00 by given conductivity.

CHAPTER II.

DISTRIBUTION FOR DIRECT-CURRENT RAILWAYS.

9. Introduction. — Electric railways almost always use the track rails for the return of current. Besides this, other differences between circuits for railways and those for power and lighting purposes are the higher voltage employed on the trolley, the greater per cent loss allowed in the line and the moving and variable nature of the loads.

Due to the track rails the calculations of railway feeders and working conductors are somewhat complex and uncertain. However, the recent character of bonding has made the calculations more reliable by giving a higher and more permanent value to the conductivity of the grounded side. In electric railways such as the open conduit and the double trolley systems, the track rails are not used for the return current, and the calculations for their circuits are therefore simpler and more definite. Where the feeders and working conductors form a complete copper circuit, the formulas on page 14 may be used.

10. Resistance of Rails. — Table 9, page 24, gives the equivalent copper section and the resistance of third rails or track rails at 20° cent., for various values of relative resistance of steel to copper. The corresponding value for any number of rails is found by multiplying the equivalent copper section, or by dividing the resistance, by the number. The resistance at any other temperature is found by multiplying the value in the table by the temperature factor of resistance for steel, T_1, Table 7, page 17, while the equivalent section of copper is equal to the tabular value divided by T_1; or, the values may be found for one rail for any other condition of relative resistance and temperature by means of the following formulas:

Equivalent cir. mils of 100 per cent copper

$$= \frac{125{,}000 \times \text{Pounds per yard}}{T_1 \times \text{Relative resistance}} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Ohms resistance per 1000 feet

$$= \frac{T_1 \times \text{Relative resistance}}{12.1 \times \text{Pounds per yard}} \quad \ldots\ldots \quad (21)$$

Ohms resistance per mile

$$= \frac{T_1 \times \text{Relative resistance}}{2.28 \times \text{Pounds per yard}} \quad \ldots\ldots \quad (22)$$

T_1 = Temperature factor of steel. Table 7, page 17.

As an example, the total resistance per mile at 30° cent. of two 65-lb. track rails having a relative resistance of 13.2 is

$$\frac{1.05 \times 13.2}{2.28 \times 65 \times 2} = 0.0468 \text{ ohm.}$$

Table 7. — Values of T_1 for Steel.

Temperature, deg. cent.	0	10	20	30	40	50
Temperature, deg. fahr.	32	50	68	86	104	122
Values of T_1	0.91	0.96	1.00	1.05	1.09	1.14

11. **Parallel Resistance of Rails and Feeders.** — The parallel resistance at any temperature of copper and aluminum feeders of any conductivity with rails of any relative resistance is easily calculated by the following method:

$$\text{Ohms resistance per 1000 feet} = \frac{10{,}350}{\frac{S_1}{T_1} + \frac{S_2}{T_2} + \frac{S_3}{T_3}} \quad \ldots\ldots \quad (23)$$

$$\text{Ohms resistance per mile} = \frac{54{,}700}{\frac{S_1}{T_1} + \frac{S_2}{T_2} + \frac{S_3}{T_3}} \quad \ldots\ldots \quad (24)$$

S_1 = Total equivalent cir. mils of copper in rails. Table 9, page 24.
S_2 = Total cir. mils in copper feeders.
S_3 = Total cir. mils in aluminum feeders.
T_1 = Temperature factor of steel. Table 7, page 17.
T_2 = Temperature factor of copper. Table 1, page 6.
T_3 = Temperature factor of aluminum. Table 1, page 6.

Thus the resistance per mile at 30° cent. of two 80-lb. rails of relative resistance of 12 in parallel with one 500,000-cir. mil

copper feeder of 97 per cent conductivity and one 500,000-cir. mil aluminum feeder of 62 per cent conductivity is

$$\frac{54{,}700}{\frac{833{,}000 \times 2}{1.05} + \frac{500{,}000}{1.069} + \frac{500{,}000}{1.674}} = 0.0232 \text{ ohm.}$$

12. **Negative Conductors.** — The track rails and negative feeders carry the return current. The size of rails is fixed by conditions other than electrical conductivity and usually give a total resistance much below that of the positive side. Electrolytic conditions may require that negative feeders be connected to the rails at certain points, but otherwise the rails generally have ample conductivity without any copper reinforcement.

The section of negative conductors may be based on a maximum allowable drop in the return. The size of the negative feeder is found by subtracting the equivalent copper section of the rails from the total circular mils required. The additional resistance of bonds may be included by increasing the true relative resistance of the rail to an apparent value. As an example of the determination of the size of negative feeder, suppose that S in formula (25) on page 26 should come out 2,000,000 cir. mils for the negative side of a circuit for which the track is to consist of two 70-lb. rails of an apparent relative resistance of 14 (including bonding). The required feeders in parallel with the track should have 2,000,000 − 625,000 × 2, or 750,000 cir. mils of copper of 100 per cent conductivity. Based on Table 8, page 20, this is equivalent to 773,000 cir. mils of copper of 97 per cent conductivity, or 1,210,000 cir. mils of aluminum of 62 per cent conductivity.

13. **Positive Conductors.** — The positive conductors consist of auxiliary feeders in parallel with trolleys or third rails. Trolley wires vary in size from Nos. 0 to 0000, while third rails usually range from 60 to 100 pounds per yard. The drop in the positive conductors is found by subtracting the calculated negative drop from the total that is allowed. Then for a third-rail system, the method is exactly like the determination of negative feeders in paragraph 12; while for systems using trolley wires, the total section of positive conductors is calculated from formula (25), page 25. The size of the auxiliary feeder is given by the difference between the total circular mils required and that of the trolley wire.

The temperature is included in the calculation of the circular mils by means of A in Table 10, page 26. For conductivities other than 100 per cent, the value of S in formula (25) is divided by the given conductivity. Thus if S comes out 1,000,000 cir. mils of 100 per cent copper, the required section of copper of 97 per cent conductivity is $\frac{1,000,000}{0.97}$, or 1,030,000 cir. mils, as may be noted from Table 8, page 20.

14. **Resistance of Circuit.**—The total resistance of the circuit is obtained by adding the resistance of the grounded side and the overhead. Where there are no negative feeders, the resistance of the rails may be taken directly from Table 9, page 24, and if no positive feeders are used, the resistance of the trolleys is given in Table 3, page 7. Thus the resistance per mile at 20° cent. of a single-track road having two 60-lb. rails of an apparent relative resistance of 14 (including bonding) and a No. 00 trolley of 100 per cent conductivity is

$$\frac{0.102}{2} + 0.411 = 0.462 \text{ ohm.}$$

15. **Given Items.**—The determination of the proper loading on which to base the feeder systems is usually more difficult than the calculation of the feeders. In general the requirement will be for a maximum drop with a very severe condition of loading or for a much smaller drop with a distributed loading of an average value. In either case the loads and their positions are first settled and then the formulas are applied successively to the separate loads; or, where the feeder system is uniform throughout, one determination is made, using the total of the loads at their center of gravity.

In case the feeder system is known the total loss is easily calculated by considering the loads separately, or by considering their combined effect where the feeder system is uniform throughout the area of loading.

For convenience in calculation the formulas have been expressed in terms of current. The per cent volt loss has also been given in terms of power, so that no preliminary approximation is necessary when the voltage is known only at the source.

16. **Examples.** — Although, in practice, these problems occur in a great many different forms, the application of the formulas will be clear from the following typical examples.

Table 8. —Equivalents of Copper of 100 Per Cent Conductivity.

Conductivity in Matthiessen's Standard Scale.					
1.00	0.99	0.98	0.97	0.96	0.62
1,000,000	1,010,000	1,020,000	1,030,000	1,040,000	1,640,000
900,000	909,000	918,000	928,000	938,000	1,450,000
800,000	808,000	816,000	825,000	834,000	1,290,000
700,000	707,000	714,000	722,000	730,000	1,130,000
600,000	606,000	612,000	618,000	625,000	984,000
500,000	505,000	510,000	515,000	521,000	820,000
400,000	404,000	408,000	412,000	417,000	656,000
300,000	303,000	306,000	309,000	312,000	492,000

For other sections, divide given circular mils of 100 per cent copper by given conductivity.
For other conductivity, divide circular mils under 1.00 by given conductivity.
To find equivalent circular mils of 100 per cent copper in a wire, multiply given circular mils by given conductivity.

Example 6. — Find the negative drop per 100 amperes at 30° cent. in a mile of track consisting of two 90-lb. rails having a relative resistance of 12.5 (including bonding) in parallel with 97 per cent copper feeders of 0, 500,000, 750,000, and 1,000,000 cir. mils, respectively.

From Table 9, page 24, equivalent cir. mils of 100 per cent copper in two rails at 20° cent. = 1,800,000.

From Table 7, page 17, temperature factor for steel at 30° cent., $T_1 = 1.05$.

From Table 1, page 6, temperature factor for 97 per cent copper at 30° cent., $T = 1.069$. Then from (24) and (28), the following values are derived:

Rails. Lb. per Yd.	Copper Feeder. Cir. Mils.	Resistance per Mile. Ohm.	Drop per Mile per 100 Amperes. Volts.
2-90	0	0.0319	3.19
2-90	1-500,000	0.0251	2.51
2-90	1-750,000	0.0226	2.26
2-90	1-1,000,000	0.0206	2.06

Owing to the comparatively large section of the rails, the combined resistance is reduced but 18 per cent for an increase

in the negative feeder from 500,000 to 1,000,000 cir. mils, equal to an increase of 100 per cent.

Example 7. — For a temperature of 40° cent., determine the line voltage at successive cars which take 100 amperes each at respective locations of 500, 2000, 3000, and 5000 feet from a power station generating 550 volts, if the circuit consists of four 80-lb. track rails having a relative resistance of 14 (including bonding), and two No. 00 trolleys of 97 per cent conductivity in parallel with one 500,000 cir. mil aluminum feeder of 62 per cent conductivity.

From Tables 7 and 9, the resistance of the tracks is,

$$\text{Resistance per 1000 feet} = \frac{0.0145 \times 1.09}{4} = 0.00395 \text{ ohm.}$$

From (23) and Tables 1 and 3, the resistance of the overhead is,

Resistance per 1000 feet

$$= \frac{10{,}350}{\dfrac{133{,}100 \times 2}{1.11} + \dfrac{500{,}000}{1.738}} = 0.01962 \text{ ohm.}$$

Hence the total resistance per 1000 feet of road is

$$0.00395 + 0.01962 = 0.02357 \text{ ohm.}$$

1st Car	— Total drop	$= 0.02357 \times 400 \times 0.5$	$= 4.7$ volts.
	— Line volts	$= 550 - 4.7$	$= 545.3$.
2d Car	— Additional drop	$= 0.02357 \times 300 \times 1.500$	$= 10.6$ volts.
	— Line volts	$= 545.3 - 10.6$	$= 534.7$.
3d Car	— Additional drop	$= 0.02357 \times 200 \times 1.000$	$= 4.7$ volts.
	— Line volts	$= 534.7 - 4.7$	$= 530$.
4th Car	— Additional drop	$= 0.02357 \times 100 \times 2.000$	$= 4.7$ volts.
	— Line volts	$= 530 - 4.7$	$= 525.3$.

In the above typical problem, the resistance of the grounded side is but 20 per cent of the resistance of the overhead.

Example 8. — A single-track road with two 75-lb. rails having a relative resistance of 13 plus 10 per cent for bonding (apparent relative resistance = 14.3), is to supply four cars with 150 amperes each when located at 0.8, 1.0, 1.5 and 2.5 miles from a substation generating 550 volts. If the minimum line e.m.f. is to be 400 volts at 50° cent., what size copper feeder of 97 per cent conductivity should be in parallel with a No. 00 trolley of 96 per cent conductivity?

From (22) and Table 7, page 17,

$$\text{Resistance per mile of two rails} = \frac{1.14 \times 14.3}{2.28 \times 75 \times 2} = 0.0477 \text{ ohm.}$$

Total drop in track $= 0.0477 \times 150\ (0.8 + 1.0 + 1.5 + 2.5) =$ 41.5 volts.

Allowable drop in overhead $= 150 - 41.5 = 108.5$ volts.

From (25), in which IL is the sum of products of each load multiplied by its distance from source, and from Table 10, page 26,

$$S = \frac{61{,}100 \times 150\ (0.8 + 1.0 + 1.5 + 2.5)}{108.5} = 490{,}000 \text{ cir. mils}$$

of 100 per cent copper.

Based on Table 8, page 20, the equivalent section of 100 per cent copper in trolley is

$$133{,}100 \times 0.96 = 128{,}000 \text{ cir. mils.}$$

Required section of 100 per cent copper in feeder

$$= 490{,}000 - 128{,}000 = 362{,}000 \text{ cir. mils.}$$

Required section of 97 per cent copper in feeder

$$= \frac{362{,}000}{0.97} = 373{,}000 \text{ cir. mils.}$$

Example 9.— Find the size of a third rail of relative resistance 7.5 (including bonding) required to start two cars taking 1000 amperes at a point midway between substations 8 miles apart, if the drop at 20° cent. is to be 25 per cent of the 600 volts at rotaries. Each track rail is to weigh 80 lb. per yard and to have a relative resistance of 13 (including bonding).

Distance from either substation to cars $= \frac{8}{2} = 4$ miles.

Current from each substation $= \frac{1000}{2} = 500$ amperes.

Total allowable drop $= 600 \times 0.25 = 150$ volts.

From Table 9, page 24, resistance per mile of two-track rails

$$= \frac{0.0713}{2} = 0.03565 \text{ ohm.}$$

Drop in track $= 0.03565 \times 500 \times 4 = 71.3$ volts.

Allowable drop in third rail $= 150 - 71.3 = 78.7$ volts.

From (25), in which A from Table 10, page 26 $= 54{,}700$,

$$S = \frac{54{,}700 \times 500 \times 4}{78.7} = 1{,}390{,}000 \text{ cir. mils.}$$

From Table 9, page 24, this section corresponds to an 85-lb. rail. Then from above and Table 9,

Total resistance per mile $= R = 0.03565 + 0.0387 = 0.07435$ ohm.

From (28), total drop $= 0.07435 \times 500 \times 4 = 148.7$ volts.

From (37), voltage at cars $= 600 - 148.7 = 451.3$ volts.

From (40), drop in per cent of volts at load is

$$V = 148.7 \div 0.01 \times 451.3 = 33.0 \text{ per cent.}$$

From (41), drop in per cent of volts at substation is

$$V_0 = 148.7 \div 0.01 \times 600 = 24.8 \text{ per cent.}$$

Watt loss. — From above, $R = 0.07435$ ohm.

From (35), $p = 0.07435 \times (500)^2 \times 8$
$= 148{,}700$ watts.

Example 10. — Calculate the size of positive feeders at 30° cent. required for a uniformly distributed load of 600 amperes per mile between two substations 6 miles apart with an average loss of 5 per cent of the 650 volts generated, if there are to be eight 100-lb. track rails and four 70-lb. third rails having relative resistances (including bonding) of 12.5 and 8.0, respectively.

Since the average loss is to be 5 per cent, the maximum is 10 per cent or 65 volts. The result is equivalent to the total load concentrated at a point one-quarter the distance between substations from either one.

Total load per substation $= \dfrac{600 \times 6}{2} = 1800$ amperes.

Center of gravity of load $= \frac{6}{4} = 2$ miles from either substation.

From Tables 7 and 9, resistance per mile of four tracks
$= 0.0548 \times 1.05 \div 8 = 0.00719$ ohm.

Drop in track $= 0.00719 \times 1800 \times 2 = 25.9$ volts.

Allowable positive drop $= 65 - 25.9 = 39.1$ volts.

From (25), $S = 57{,}400 \times 1800 \times 2 \div 39.1 = 5{,}280{,}000$ cir. mils of 100 per cent copper.

From Tables 7 and 9, third-rail section $= 1{,}090{,}000 \times 4 \div 1.05$
$= 4{,}150{,}000$ cir. mils of 100 per cent copper.

Required section of 100 per cent copper feeder
$= 5{,}280{,}000 - 4{,}150{,}000 = 1{,}130{,}000$ cir. mils.

Required section of 97 per cent copper feeder
$= 1{,}130{,}000 \div 0.97 = 1{,}165{,}000$ cir. mils.

Required section of 62 per cent aluminum feeder
$= 1{,}130{,}000 \div 0.62 = 1{,}823{,}000$ cir. mils.

Table 9. — Resistance to Direct-Current of One Steel Rail at 20° Cent. or 68° Fahr.

Pounds per Yard.	Relative Resistance of Steel to Copper.															
	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0
	Equivalent Circular Mils of Copper of 100 Per Cent Matthiessen's Standard.															
100	1920000	1780000	1670000	1560000	1470000	1390000	1310000	1250000	1190000	1140000	1090000	1040000	1000000	961000	926000	892000
95	1830000	1700000	1580000	1480000	1400000	1320000	1250000	1190000	1130000	1080000	1030000	989000	950000	914000	880000	846000
90	1730000	1610000	1500000	1400000	1320000	1250000	1180000	1120000	1070000	1020000	978000	937000	900000	865000	833000	803000
85	1630000	1520000	1410000	1330000	1250000	1180000	1120000	1060000	1010000	965000	924000	885000	850000	817000	787000	759000
80	1540000	1430000	1330000	1250000	1180000	1110000	1050000	1000000	951000	908000	869000	833000	800000	769000	740000	714000
75	1440000	1340000	1250000	1170000	1100000	1040000	988000	936000	893000	857000	815000	781000	750000	721000	694000	669000
70	1340000	1250000	1170000	1090000	1030000	971000	920000	875000	832000	795000	760000	729000	700000	673000	648000	625000
60	1150000	1070000	1000000	938000	882000	832000	789000	750000	713000	681000	652000	625000	600000	577000	555000	535000
	Resistance per 1000 Feet of Continuous Rail, in Ohms.															
100	0.00540	0.00582	0.00623	0.00664	0.00705	0.00748	0.00789	0.00831	0.00872	0.00914	0.00955	0.00997	0.0104	0.0108	0.0112	0.0116
95	0.00568	0.00612	0.00656	0.00700	0.00743	0.00788	0.00831	0.00875	0.00920	0.00963	0.0101	0.0105	0.0109	0.0114	0.0118	0.0122
90	0.00600	0.00646	0.00692	0.00738	0.00785	0.00831	0.00877	0.00923	0.00970	0.0102	0.0106	0.0111	0.0115	0.0120	0.0125	0.0129
85	0.00635	0.00685	0.00733	0.00782	0.00831	0.00880	0.00929	0.00978	0.0103	0.0108	0.0112	0.0117	0.0122	0.0127	0.0132	0.0137
80	0.00675	0.00728	0.00779	0.00831	0.00883	0.00935	0.00988	0.0104	0.0109	0.0114	0.0119	0.0125	0.0130	0.0135	0.0140	0.0145
75	0.00720	0.00775	0.00831	0.00886	0.00942	0.00997	0.0105	0.0111	0.0116	0.0122	0.0127	0.0133	0.0138	0.0144	0.0150	0.0155
70	0.00771	0.00831	0.00890	0.00950	0.0101	0.0107	0.0113	0.0119	0.0125	0.0131	0.0136	0.0142	0.0148	0.0154	0.0160	0.0166
60	0.00900	0.00970	0.0104	0.0111	0.0118	0.0125	0.0132	0.0138	0.0145	0.0152	0.0159	0.0166	0.0173	0.0180	0.0187	0.0194
	Resistance per Mile of Continuous Rail, in Ohms.															
100	0.0285	0.0307	0.0328	0.0350	0.0372	0.0384	0.0417	0.0438	0.0460	0.0482	0.0504	0.0526	0.0548	0.0570	0.0593	0.0613
95	0.0300	0.0323	0.0346	0.0369	0.392	0.0415	0.0438	0.0462	0.0485	0.0508	0.0531	0.0554	0.0577	0.0600	0.0623	0.0647
90	0.0317	0.0341	0.0365	0.0390	0.0414	0.0438	0.0463	0.0487	0.0512	0.0536	0.0560	0.0585	0.0609	0.0633	0.0658	0.0682
85	0.0335	0.0361	0.0387	0.0413	0.0438	0.0464	0.0490	0.0516	0.0542	0.0568	0.0593	0.0619	0.0645	0.0670	0.0697	0.0722
80	0.0356	0.0384	0.0411	0.0438	0.0466	0.0493	0.0520	0.0548	0.0576	0.0603	0.0630	0.0658	0.0685	0.0713	0.0741	0.0768
75	0.0380	0.0408	0.0438	0.0467	0.0497	0.0526	0.0555	0.0584	0.0614	0.0643	0.0672	0.0702	0.0731	0.0760	0.0790	0.0819
70	0.0407	0.0438	0.0470	0.0501	0.0532	0.0563	0.0595	0.0626	0.0657	0.0689	0.0720	0.0751	0.0783	0.0814	0.0847	0.0877
60	0.0475	0.0511	0.0548	0.0584	0.0621	0.0658	0.0694	0.0730	0.0767	0.0803	0.0840	0.0877	0.0914	0.0950	0.0988	0.102

For other temperature, divide circular mils and multiply resistance by T_1, Table 7, page 17.
For any number of rails, multiply circular mils and divide resistance by the number.

Formulas for D. C. Railway Circuits.

Required Items.	Positive or Negative Alone.	Positive and Negative Total.
Circular mils or Total resistance.	*A* from Table 10, page 26. $S = \frac{AIL}{v}$ . . (25)	$R = \frac{v}{IL}$ . . . (26)
Volt drop.	$v = \frac{AIL}{S}$ . . (27)	See Par. 14, page 19. $v = RIL$. . . (28)
Per cent volt drop or Per cent power loss from amperes.	$V = \frac{AIL}{0.01\,Se}$. (29)	$V = \frac{RIL}{0.01\,Se}$. (30)
	$V_0 = \frac{AIL}{0.01\,Se_0}$. (31)	$V_0 = \frac{RIL}{0.01\,Se_0}$. (32)
Per cent volt drop from watts at load.	$V_0''' = \frac{AwL}{0.01\,Se_0^2}$. (33)	$V_0''' = \frac{RwL}{0.01\,Se_0^2}$. (34)
	V_0 from Table 31, page 72.	
	T from Table 1 or 7. *R* from Table 3, page 7.	
Watt loss.	$p = TRI^2L = 0.01Vw = \frac{0.01V_0w}{1 - 0.01V_0}$ . . (35)	
Volts at source.	$e_0 = e + v = e(1 + 0.01V) = \frac{e}{1 - 0.01V_0}$. (36)	
Volts at load.	$e = e_0 - v = \frac{e_0}{1 + 0.01V} = e_0(1 - 0.01V_0)$. (37)	
Watts at source.	$w_0 = w + p = w(1 + 0.01V) = \frac{w}{1 - 0.01V_0}$. (38)	
Amperes.	$I = \frac{w}{e} = \frac{w}{e_0(1 - 0.01V_0)} = \frac{w_0}{e_0}$ . . . . . (39)	

When given V, find v from $v = 0.01Ve = 0.01Ve_0 / (1 + 0.01V)$ . . . (40)
When given V_0, find v from $v = 0.01V_0e_0 = 0.01V_0e / (1 - 0.01V_0)$ . . (41)
When given w, find I from (39) above.

NOTATION.

A = Resistance factor. Table 10, page 26.
e = Volts at load.
e_0 = Volts at source.
I = Total current at source, in amperes.
L = Distance from source to load, in miles or 1000 feet.
p = Power loss in watts.
R = Ohms resistance per mile for L in miles. / Ohms resistance per 1000 feet for L in 1000 feet.
S = Equivalent cir. mils of copper of 100 per cent conductivity.
T = Temperature factor. Table 1, page 6, or Table 7, page 17.
V = Volt loss in per cent of volts at load.
V_0 = Volt loss in per cent of volts at source.
V_0''' = Volt loss factor. Table 31, page 72.
v = Volt drop.
w = Watts at load.
w_0 = Watts at source.

Table 10.—Values of *A* for Wires and Rails.

Temperature in degrees.		Wires.		Rails. With or without feeders.	
Cent	Fahr.	For L in. Miles.	For L in. 1000 Ft	For L in. Miles.	For L in. 1000 Ft.
0	32	50,600	9,580	49,800	9,420
10	50	52,700	9,970	52,500	9,950
20	68	54,700	10,350	54,700	10,350
30	86	56,800	10,740	57,400	10,870
40	104	58,900	11,140	59,600	11,280
50	122	61,100	11,560	62,400	11,800

The above values for wires are for copper of 100 per cent conductivity in Matthiessen's standard scale.

The above values for rails are for equivalent circular mils of copper of 100 per cent conductivity in Matthiessen's standard scale.

PART II.

ALTERNATING-CURRENT TRANSMISSION BY MEANS OF OVERHEAD, UNDERGROUND AND INTERIOR WIRES FOR PURPOSES OF LIGHT, POWER AND TRACTION

ALTERNATING-CURRENT TRANSMISSION

CHAPTER III.

ALTERNATING-CURRENT TRANSMISSION BY OVERHEAD WIRES

17. Introduction. — The methods in common use for calculating alternating-current transmission lines from the volt loss are either indirect or inaccurate. This condition results from the fact that it is impossible to devise an exact formula for any alternating-current system which will directly indicate the size of wire required for the transmission of a given amount of power with a given volt loss. Methods of the indirect class require that the size wire be known, and hence are trial methods. Approximations of the second class may be sufficiently close under certain conditions but give wholly erroneous results for other practical cases.

The essential feature of the desirable method is that it should directly determine for any system and frequency, the size of wire, power loss and power-factor at generator when given the volt loss, length of line, power received, load power-factor, voltage at generator or load and distance between wires. The following method accomplishes this result with commercial accuracy over a sufficiently wide range of conditions to cover almost all practical cases.

One familiar method of calculating alternating-current lines assumes that the impedance volts equals the line loss, or in Figs. 1 and 2 $AB = AC$. Several other methods in common use assume that the line loss equals the projection of the impedance volts on the vector of delivered voltage. The errors of these approximations in terms of the actual volt loss are shown in Table 11, for copper wires on 36-inch centers and a true volt loss of 10 per cent of the generated volts.

The errors for leading power-factors, larger wires or greater spacings exceed those shown in Table 11, and it is evident then, that both methods may lead to erroneous results. The

first assumption gives wires which are too large, while the other assumption gives wires too small. By stating the errors of Table 11 in terms of the calculated values, the percentages shown would be greatly decreased for the first assumption and similarly increased in the second case.

18. Outline of Method. — All items which depend on the frequency, size of wire, load power-factor and wire spacing were grouped into one term, which is denoted by M and called the wire factor. All other variables that determine the size of wire

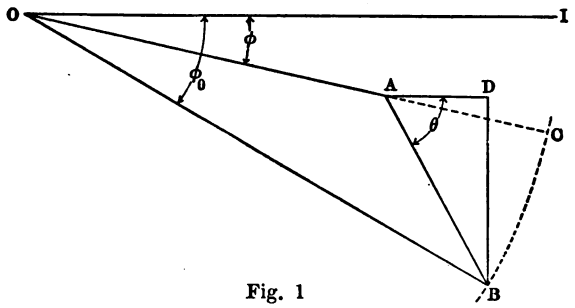

Fig. 1

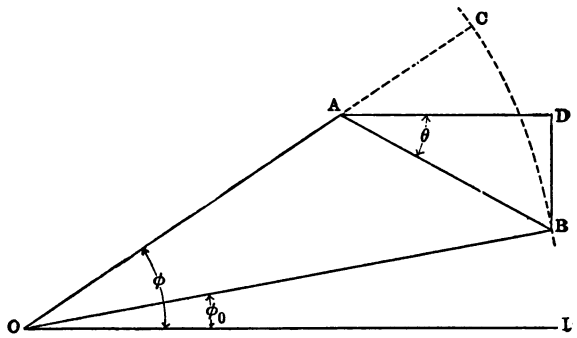

Fig. 2

were grouped into the second member of the equation of M. Then after introducing the proper transmission factors, the equation took the form shown in the table of formulas opposite the size of wire. The remaining formulas are simple derivations either from the equation of M or from well known relations.

The vector diagram from which the fundamental equation was derived is shown in Fig. 1 for lagging current and in Fig. 2 for leading current, wherein,

OI = Current at load. OB = Volts at source.
OA = Volts at load. $AC = OB - OA$ = Volt loss.
AD = Resistance volts. ϕ = Power-factor angle of load.
DB = Reactance volts. θ = Power-factor angle of line.
AB = Impedance volts. ϕ_0 = Power-factor angle at source.

19. Range of Application. — The formulas apply for all volt losses less than 20 per cent of the generated voltage or 25 per cent of the voltage at load, and for a sufficient range of wire sizes to cover the usual and unusual cases in practice. At high standard frequencies and for power-factors near 100 per cent the values of M for the largest wires were omitted in order to confine the maximum possible error within the limits prescribed in Table 12, page 32.

Table 11. — Error in Per Cent of True Volt Loss.

Size of Wire. A.W.G.	25 Cycles per Sec.			60 Cycles per Sec.			125 Cycles per Sec.		
	Lagging Power Factor of Load in Per Cent.								
	100	90	80	100	90	80	100	90	80
Assuming Impedance Volts = Volt Loss.									
0000	36	4	0	110	29	13	200	55	29
0	11	0	0	53	10	2	125	33	16
4	1	1	7	12	0	0	44	7	1
Assuming Projection of Impedance Volts = Volt Loss.									
0000	5	0	0	19	4	1	43	8	4
0	1	0	0	8	1	0	22	4	1
4	0	0	0	1	0	0	6	0	0

20. Maximum Error. — As previously stated, any direct method must be approximate. However, all errors introduced by the following method are easily within conservative limits and usually negligible. In determining the size of wire from the volt loss the maximum error is about 10 per cent of the wire factor M, which is usually less than the difference of M for successive wires in commercial sizes. The maximum error occurs near 0 per cent and 20 per cent volt losses but gradually

reduces to a zero error near 10 per cent loss. The point where the formula for M should be correct was arbitrarily fixed at 10 per cent volt loss, since this is about the mean value in practice.

After the wire is determined, the actual per cent drop is calculated. In order to minimize its error and that of the remaining items, each column in the tables of M has been divided into a maximum of three sections. It will be observed that any value of M is found between the section letters a and b, b and c, or c and d, depending upon the size of wire, frequency, spacing and power-factor.

The maximum errors in the calculations are shown in Table 12 below. These errors occur near 0 per cent and 20 per cent volt losses but gradually reduce to zero near 10 per cent loss. In practical problems the calculated value of M is most often between the section letters c and d, and therefore the errors of the remaining calculations are negligible.

It should be observed that the values in Table 12 are expressed in per cent of the true result. Thus for an error of 2 per cent with a true volt loss of 5 per cent, the calculated value would be no greater than 5.1 and no less than 4.9 per cent.

Table 12. — Maximum Errors in Per Cent of True Values at 20° Cent.

Calculated Items.	For M in Section		
	a--b.	b—c.	c—d.
Volt loss or per cent volt loss	6.0	4.0	2.0
Kilovolts at load or source	1.5	1.0	0.5
Power loss or kilowatts at source, when given E	0.0	0.0	0.0
Power loss or per cent power loss, when given E_0	3.0	2.0	1.0
Kilowatts at source, when given E_0	0.8	0.5	0.3
Amperes per wire, when given E	0.0	0.0	0.0
Amperes per wire, when given E_0	1.5	1.0	0.5
Power-factor at source, when given E	1.5	1.0	0.5
Power-factor at source, when given E_0	2.3	1.5	0.8

21. **Transmission Systems.** — Systems for alternating-current transmission are either one, two or three-phase. These results are applicable to one and three-phase systems, and to the two-phase system with either three or four wires of equal size.

Accurate formulas cannot be given for the two-phase three-wire system where the size of the common wire is different from the others. However, the result may be approximated by calculating for three equal wires and then making the proper allowance for the larger cross section of the common lead. (See Example 16, page 44.) In the three-phase four-wire system the neutral wire carries no current when the system is balanced, and hence the results are exactly similar to those for the three-phase system with three wires.

22. Balanced Loads. — A balanced load is assumed in all the formulas. In practice such is often not the case, although the variation from a balanced condition is usually small. The effect of unbalancing is to alter the voltage in proportion to the amount of unbalancing. The other items are also affected, but seldom is the discrepancy of any practical importance.

23. Temperature. — All results have been calculated for a temperature of 20° cent. or 68° fahr. The value of M varies directly with the temperature, but it also depends to a lesser degree on other conditions. However, as the error of commission is much less than the error of omission, the values of A in Table 16, page 52, have been calculated for various temperatures.

24. Specific Conductivity. — The calculations have been made for a specific conductivity of 100 per cent for copper and 62 per cent for aluminum. The value of M varies inversely with the specific conductivity but also depends somewhat upon other conditions. However, sufficiently accurate results are obtained by including the proper conductivity of the conductor by the method shown in Table 16.

25. Solid and Stranded Conductors. — Wires smaller than No. 0 have been considered solid, while the larger sizes have been taken stranded. However, since the inductance of a stranded wire is between that of a solid wire of the same cross section of metal and one of the same diameter, the discrepancy in the final results for any ordinary variation from the assumed conditions is not appreciable.

26. Skin Effect. — Owing to a decreasing current density toward the center of wires carrying alternating currents, the effective resistance is increased in direct proportion to the product of its cross section and the frequency. In the usual sizes of transmission wire the effect is negligible, and even in the

largest sizes shown for the higher frequencies the maximum additional resistance is less than 5 per cent, which value decreases very rapidly with the size of wire.

27. **Wire Spacing.** — Inductance increases directly with the distance from center to center of the wires. However, the effect on the impedance of the line for large variations in the spacing is not great, even at the higher commercial frequencies. In order to cover all cases in practice, the wire factor has been calculated for three different spacings at each frequency. In general the results for wires on 18 and 60-inch centers will be sufficiently accurate for spacings less than 27 inches and greater than 48 inches, respectively, while the values for 36 inches will cover spacings from 27 to 48 inches. But, where greater accuracy is desired for spacings other than shown, the values of M may be readily interpolated from the tables.

28. **Arrangement of Wires.** — It is assumed in the two and three-phase systems with three wires that the conductors are placed at the three corners of an equilateral triangle. For other arrangements, with the wires properly transposed, little error is introduced by taking the distance from center to center of the wires as the average of the distances between wires 1 and 2, 2 and 3, and 1 and 3. In the two-phase four-wire system the distance between wires of the same phase should be used.

29. **Frequency.** — The tables have been calculated for all standard frequencies. The values for 15 cycles per second are necessary for the design of single-phase railway systems, while 25, 40, 60, and 125 cycles per second cover the remaining systems of transmission for purposes of light, power, and traction. However, where an odd frequency is to be employed, the required value of M may be interpolated from the tables.

30. **Multiple Circuits.** — Where circuits are in parallel from the source to receiver, the load should be proportioned between them and each line calculated separately.

31. **Current-carrying Capacity.** — The current-carrying capacity in amperes per wire is shown in Table 4, page 8, for both copper and aluminum. The values in the table are based on a temperature rise of 40° cent. or 72° fahr., but the current-carrying capacity for any temperature elevation may be found from formula (4), page 9.

In general, overhead transmission lines are of sufficient length

to insure the necessary current-carrying capacity when the conditions of volt loss are met. However, it is desirable to note from Table 4 that such is the case, after each determination of wire.

32. **Transmission Voltage.** — The voltage is taken between wires, either at the load or source. The term "source" designates the generator, the secondary terminals of step-up or step-down transformers, or a certain point of the circuit from which the calculation is made. In general the voltage at the source is given, although in some cases of transmission to a single center of distribution, the voltage at the load is specified. The formulas have been stated in terms of both voltages in order to cover all cases without any preliminary approximation. For convenience the voltage has been expressed in kilovolts, thousands of volts. In a two-phase four-wire system the voltage between the wires of the same phase is specified.

With lagging current the voltage at the load is always less than the voltage at the generator. With leading current the voltage at load is less when the sum of the power-factor angles of load and line is less than 90 degrees, but it gradually becomes greater than the voltage at the generator as their sum increases above 90 degrees.

33. **Volt Loss.** — In an alternating-current system the volt loss in the line is the difference between the voltages at the generator and at the load. The line loss is always less than the impedance volts and almost always greater than the projection of the impedance volts on the vector of delivered volts. It may be greater or less than the resistance volts. (See Figs. 1 and 2, page 30.)

The per cent volt loss may be expressed in terms of the volts at load or source. However, either may be obtained from the other by means of the simple equations shown below the table of formulas on page 50. For convenience in calculations the per cent loss is expressed as a whole number.

34. **Power Transmitted.** — The power transmitted is always specified at the load, and is usually expressed as true power in kilowatts or as apparent power in kilovolt-amperes. Where the load is given in amperes the equivalent value of kilowatts may be obtained from equations (59) or (60) solved for W. The values of B in Table 17, page 52, serve to make the formula

for amperes per wire applicable to all systems of transmission.

For balanced loads in one-phase, two-phase four-wire or three-phase circuits the current is the same in each wire, while in the two-phase three-wire system, the current in the common lead is 1.41 times that in each of the other wires. In the two-phase three-wire system I represents the amperes in each of the outer wires, this being the same as the current per phase.

35. **Power Loss.** — The power loss in any system depends only upon the current and resistance. The per cent power loss may be greater or less than the per cent volt loss. It is less than the per cent volt loss, V, when the power-factor at the source is less than the power-factor of the load, and is greater when the reverse is true. An exception to this statement may occur when capacity effects in the line are included. (See Example 20, page 48, and Example 25, page 69.) The power at the source is the sum of the power at load and the power loss in line.

36. **Power-Factor.** — The power-factor of the load is always given. The values of M have been calculated from 95 per cent lead to 75 per cent lag, for all frequencies except 125 cycles per second, giving a range sufficient to cover almost all practical cases of transmission. The power-factor at the load should be known accurately, as the value of M varies greatly with it.

The power-factor at the source depends upon the power-factor at the load, the per cent power loss and the per cent volt loss. For current leading at the load, the power-factor is less leading at the source than at the load, and even may become lagging at the source. (See Example 24, page 68.) Lagging current at the load is always accompanied by lagging current at the source. The power-factor at the source is lower than the power-factor of the load when the volt loss, V, is a larger percentage than the power loss, P, and it is higher at the source when the reverse is true. This statement may be modified by capacity effects in the line. (See Example 20, page 48, and Example 25, page 69.)

37. **Wire Factor.** — The wire factor, M, depends upon the impedance of the wires, and on the power-factor angles of the load and line. The values have been calculated for copper and aluminum at all standard frequencies, for a range of sizes and load power-factors sufficient to cover all practical cases likely

to arise in transmission design. Tables 18 to 22 give the results for copper, and Tables 23 to 27 for aluminum. Those values which, under certain conditions, might lead to errors greater than previously specified have been omitted.

38. Given Items. — The given problem may be of two kinds; either the line loss is specified and the size of wire is required, or the size of wire is specified and the line loss is required. In both cases the additional items to be given are system of transmission, conductor metal, temperature, frequency, wire spacing, distance of transmission, power delivered, power-factor of load, and the voltage between wires at the source of load.

39. Size of Wire. — The size of wire is generally determined from the per cent volt loss, although in some instances it is fixed by the condition of per cent power loss. Formulas are given for both methods of procedure.

When the per cent volt loss is given, the value of A is taken from Table 16, page 52, for the given system of transmission, temperature and specific conductivity. From Table 15, page 51, V' or V_0' is found, depending upon whether the voltage is given at the load or source. The value of M is then calculated from formula (46) or (47), and the size of wire is noted from Tables 18 to 27, opposite the required value of M.

When given the per cent power loss, R is calculated from formula (48) or (49), and the size of wire is found in Table 3, page 7, opposite the required resistance per mile. The wire found by either method is the required size of each conductor.

40. Per Cent Volt Loss. — It is desirable to calculate the per cent volt loss after the size of wire is determined. The value of M whether between the section letters a and b, b and c, or c and d, is noted from the proper table. The value of V'' or V_0'' is calculated from formula (50) or (51), and then located in Table 15, page 51, in the column headed by the same section letters as noted above for M. The corresponding value of per cent volt loss in terms of the volts at load or source, respectively, is then obtained in the column headed by V or V_0. Where the calculated value does not appear in the table, the corresponding value of V or V_0 is easily found by interpolation.

The remaining calculations are clearly outlined in the table of formulas on page 50. When the voltages at the load and source become known, either formula for any required item may be used.

41. Charging Current. — The total charging current for a single-phase circuit or for each phase of a two-phase four-wire circuit is

$$\frac{0.000122\ E_0 fL}{\log_{10} \frac{2\ D}{d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

The charging current per wire of a three-phase three-wire circuit is

$$\frac{0.000141\ E_0 fL}{\log_{10} \frac{2\ D}{d}} \quad . \quad (43)$$

E_0 = Kilovolts between wires at source.
f = Frequency in cycles per second.
L = Distance from source to load, in miles.
D = Distance between center of wires.
d = Diameter of wire in same unit as D.

42. Capacity Effects. — Capacity influences the voltage loss, the power loss and the power-factor at generator. In all systems except those of unusual length or high commercial frequency, capacity may be entirely neglected without any disturbing error. However, the effect of capacity may be easily included in the calculations by assuming that the same result is produced by substituting for the distributed capacity of the line one-half its total at each end.* The true power-factor of the load is then replaced by an apparent value determined as follows:

$$t' = t - \frac{cE^2fL}{W} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

After t' has been calculated, the corresponding apparent power-factor is taken from Table 14, page 39, and this value is used in the table of M for the determination of the size of wire, volt loss and power loss.

The true power-factor at the source is obtained by the following method: Formula (61) is solved for K_0', using the apparent power-factor of load for K. The value of t_0' corresponding to K_0' is noted from Table 14, page 39, and substituted in formula (45) for the reactance factor. The true power-factor at source is then obtained from Table 14, by finding K_0 corresponding to t_0. (See Example 20, page 48, and Example 25, page 69.)

$$t_0 = t_0' - \frac{cE_0^2fL}{W_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

* Method of H. Pender, Proceedings of A.I.E.E., June, 1908, page 772.

In formulas (44) and (45) above,

c = Capacity factor. Table 13, page 39.
E = Kilovolts between wires at load.
E_0 = Kilovolts between wires at source.
f = Frequency in cycles per second.
L = Distance from source to load, in miles.
t = Reactance factor for true power-factor of load. Table 14.
t' = Reactance factor for apparent power factor of load.
t_0 = Reactance factor for true power-factor at source.
t_0' = Reactance factor for apparent power-factor at source.
W = Kilowatts at load.
W_0 = Kilowatts at source.

Table 13. — Values of c for Overhead Wires.

Cir. Mils or A.W.G.	1-Phase System, or 2-Phase 4-Wire System			3-Phase 3-Wire System.		
	Distance Between Center of Wires, in Inches.					
	18	36	60	18	36	60
1,000,000 to 500,000	0.00004	0.00003	0.00003	0.00008	0.00007	0.00006
500,000 to 0000	0.00004	0.00003	0.00003	0.00007	0.00006	0.00006
0000 to 8	0.00003	0.00003	0.00002	0.00006	0.00005	0.00005

Table 14. — Reactance Factors.

Power-factors in per cent.	95 Lead.	98 Lead.	100	98 Lag.	95 Lag.	90 Lag.	85 Lag.	80 Lag.	75 Lag.
Reactance factors.....	-0.33	-0.20	0	0.20	0.33	0.49	0.62	0.75	0.88

43. **Examples.** — Examples illustrating the application of the formulas on page 50 have been chosen at random and are solved below in detail. The directness, simplicity, range and accuracy of the method will at once be apparent. As a comparison some of the examples are based on problems already in print, although in most cases direct solutions are impossible by other methods. In order to prove the accuracy of the method, the exact result

has been inclosed in brackets after many of the calculated values.

Example 11. — A transformer substation of an electric railway is to be supplied with 3200 kw. at 15 cycles per second and 80 per cent lagging power-factor over 40 miles of single-phase copper line with its wires on 60-inch centers. The loss is to be 15 per cent of the 33,000 volts generated, the temperature 20° cent., and the wires are to be of 100 per cent conductivity.

GIVEN ITEMS.

$W = 3200$ kw.; $E_0 = 33$ kv.; $V_0 = 15$ per cent;
$L = 40$ miles; $K = 0.80$ lag.

From Table 16, page 52, $A = 0.200$.
From Table 15, page 51, $V_0' = 10.9$.

REQUIRED ITEMS.

Size of each wire. — From (47),

$$M = \frac{(33)^2 \times 10.9}{0.2 \times 3200 \times 40} = 0.464.$$

From Table 18, page 53, use No. 00, for which $M = 0.494$ $(c - d)$.

Per cent volt loss. — From (51),

$$V_0'' = \frac{0.494 \times 0.2 \times 3200 \times 40}{(33)^2} = 11.6\ (c-d).$$

From Table 15, page 51,

$V_0 = 15.3$ per cent. [15.3 per cent.]

Kilovolts at load. — From (53),

$E = 33\ (1 - 0.153) = 28.0$ kv. [28.0 kv.]

Volt loss. — From (54),

$v = 1000\ (33 - 28) = 5000$ volts. [5000 volts.]

Per cent power loss. — From Table 3, page 7,

$R = 0.411$ ohm.

From (56), $P = \dfrac{0.411 \times 0.2 \times 3200 \times 40}{[33\ (1 - 0.153)\ 0.80]^2} = 21.0$ per cent.

[21.0 per cent.]

Kilowatts at source. — From (57),

$W_0 = 3200\ (1 + 0.21) = 3870$ kw. [3870 kw.]

Watt loss. — From (58),

$$P = 1000\ (3870 - 3200) = 670{,}000 \text{ watts.} \quad [670{,}000 \text{ watts.}]$$

Amperes per wire. — From Table 17, page 52, $B = 1.000$.
From (60),

$$I = \frac{3200}{33\ (1 - 0.153)\ 0.80} = 143 \text{ amperes.} \quad [143 \text{ amperes.}]$$

From Table 4, page 8, No. 00 conductors will safely carry this current.

Power-factor at source. — From (61),

$$K_0 = (1 + 0.21)\ (1 - 0.153)\ 0.80 = 0.820. \quad [0.820.]$$

Example 12. — A lighting transformer 10,000 feet distant is to receive 50 kw. at 95 per cent lagging power-factor and 125 cycles per second with a loss of 5 per cent of the 2000 volts at load at a temperature of 30° cent. The line is to be single-phase and of copper conductors of 100 per cent conductivity on 18-inch centers.

GIVEN ITEMS.

$W = 50$ kw.; $E = 2$ kv.; $V = 5$ per cent;
$L = 10{,}000$ feet; $K = 0.95$ lag.

From Table 16, page 52, $A = 0.0393$.
From Table 15, page 51, $V' = 4.8$.

REQUIRED ITEMS.

Size of each wire. — From (46),

$$M = \frac{(2)^2 \times 4.8}{0.0393 \times 50 \times 10} = 0.977.$$

From Table 22, use No. 0, for which $M = 0.905\ (b - c)$.
Per cent volt loss. — From (50),

$$V'' = \frac{0.905 \times 0.0393 \times 50 \times 10}{(2)^2} = 4.45\ (b - c).$$

From Table 15, page 51, $V = 4.75$ per cent. [4.75 per cent.]
Kilovolts at source. — From (52),

$$E_0 = 2\ (1 + 0.0475) = 2.10 \text{ kv.} \quad [2.10 \text{ kv.}]$$

Per cent power loss. — From Table 3, page 7, $R = 0.518$ ohm.

From (55),

$$P = \frac{.518 \times .0393 \times 50 \times 10}{(2)^2 \times (0.95)^2} = 2.82 \text{ per cent.} \quad [2.82 \text{ per cent.}]$$

Kilowatts at source.—From (57),

$$W_0 = 50\ (1 + 0.0282) = 51.4 \text{ kw.} \qquad [51.4 \text{ kw.}]$$

Amperes per wire.—From Table 17, page 52, $A = 1.000$.

From (59), $$I = \frac{50}{2 \times 0.95} = 26.3 \text{ amperes.} \qquad [26.3 \text{ amperes.}]$$

Power-factor at source.—From (61),

$$K_0 = \frac{(1 + 0.0282)\ 0.95}{1 + 0.0475} = 0.932. \qquad [0.932.]$$

Example 13. — (From *Electric Journal*, 1907, page 231, Ex. 2.)

GIVEN ITEMS.

$W = 75$ kw.; $E = 2$ kv.; Wires = No. 4;
$L = 5000$ feet; $K = 0.95$ lag.

System: 1-phase, 60 cycles, 18-inch spacing.

Assume wires of 100 per cent copper and temperature of 20° cent.

From Table 16, page 52, $A = 0.0379$.

From Table 21, page 56, $M = 1.38\ (c - d)$.

REQUIRED ITEMS.

Per cent volt loss.—From (50),

$$V'' = \frac{1.38 \times 0.0379 \times 75 \times 5}{(2)^2} = 4.90\ (c - d).$$

From Table 15, page 51,

$$V = 5.40 \text{ per cent.} \qquad [5.40 \text{ per cent.}]$$

Kilovolts at source.—From (52),

$$E = 2\ (1 + 0.054) = 2.11 \text{ kv.} \qquad [2.11 \text{ kv.}]$$

Volt loss.—From (54),

$$v = 1000\ (2.11 - 2.00) = 110 \text{ volts.} \qquad [110 \text{ volts.}]$$

Example 14. — (From "Alternating Currents" by Franklin and Esty, 1907, page 321.)

GIVEN ITEMS.

$W = 1000$ kw.; $E_0 = 23$ kv.; $E = 20$ kv.;
$L = 30$ miles; $K = 0.85$ lag.

System: 1-phase, 60 cycles, 18-inch spacing.

Assume temperature of 20° cent. and copper of 100 per cent conductivity.

From Table 16, page 52, $A = 0.200$.

$V = \frac{3}{0.20} = 15$ per cent. From Table 15, page 51, $V' = 13.1$.

REQUIRED ITEMS

Size of each wire. — From (46),

$$M = \frac{(20)^2 \times 13.1}{0.2 \times 1000 \times 30} = 0.873.$$

From Table 21, page 56, use No. 1, for which

$$M = 0.936\,(c - d).$$

Per cent volt loss. — From (50),

$$V'' = \frac{0.936 \times 0.2 \times 1000 \times 30}{(20)^2} = 14.0\,(c - d).$$

From Table 15, page 51, $V = 15.75$ per cent. [15.5 per cent.]

Example 15. — A load of 750 kw. of 2-phase power at 25 cycles per second and 100 per cent power-factor is to be delivered over 5 miles of aluminum line of 4 wires with 36 inches between conductors of the same phase, with a loss of 7.5 per cent of the 6600 volts generated, at 30° cent.

GIVEN ITEMS.

$W = 750$ kw.; $E_0 = 6.6$ kv.; $V_0 = 7.5$ per cent;
$L = 5$ miles; $K = 1.00$.

From Table 16, page 52, $A = 0.104$.
From Table 15, page 51, $V_0' = 6.4$.

REQUIRED ITEMS.

Size of each wire. — From (47),

$$M = \frac{(6.6)^2 \times 6.4}{0.104 \times 750 \times 5} = 0.715.$$

From Table 23, page 58, use No. 0, for which

$$M = 0.755\ (c - d).$$

Per cent volt loss. — From (51),

$$V_0'' = \frac{0.755 \times 0.104 \times 75 \times 5}{(6.6)^2} = 6.76\ (c - d).$$

From Table 15, page 51,

$$V_0 = 8.16 \text{ per cent.} \qquad [8.10 \text{ per cent.}]$$

Kilovolts at load. — From (53),

$$E = 6.6\ (1 - 0.0816) = 6.06 \text{ kv.} \qquad [6.07 \text{ kv.}]$$

Per cent power loss. — From Table 3, page 7, $R = 0.837$ ohm.

From (55),

$$P = \frac{0.837 \times 0.104 \times 750 \times 5}{(6.06)^2} = 8.89 \text{ per cent.} \qquad [8.82 \text{ per cent.}]$$

Kilowatts at source. — From (57),

$$W = 750\ (1 + 0.0889) = 817 \text{ kw.} \qquad [816 \text{ kw.}]$$

Amperes per wire. — From Table 17, page 52, $B = 0.500$.

From (59), $$I = \frac{0.5 \times 750}{6.06} = 61.9 \text{ amperes.} \qquad [61.8 \text{ amperes.}]$$

From Table 4, page 8, No. 0 aluminum conductors will easily carry this current.

Power-factor at source. — From (61),

$$K_0 = (1 + 0.0889)\ (1 - 0.0816) = 1.00. \qquad [1.00.]$$

Example 16. — A two-phase three-wire line 30 miles long is to deliver 2000 kw. at 40 cycles per second and 98 per cent lagging power-factor with a loss of 10 per cent of the 20,000 volts at load. Calculate the size of copper wires of 98 per cent conductivity on 36-inch centers for a temperature of 50° cent.

GIVEN ITEMS.

$W = 2000$ kw.; $E = 20$ kv.; $V = 10$ per cent;
$L = 30$ miles; $K = 0.98$ lag.

From Table 16, page 52, $A = \frac{0.135}{0.98} = 0.138$.

From Table 15, page 51, $V' = 9.1$.

REQUIRED ITEMS.

Size of each wire. — From (46),

$$M = \frac{(20)^2 \times 9.1}{0.138 \times 2000 \times 30} = 0.440.$$

From Table 20, page 55, use three No. 00 wires, or increase the common lead to No. 000. For No. 00, $M = 0.466 \quad (c - d)$.

Per cent volt loss. — From (50),

$$V'' = \frac{0.466 \times 0.138 \times 2000 \times 30}{(20)^2} = 9.64\ (c - d).$$

From Table 15, page 51, $V = 10.84$ per cent.

Since the volt loss in the common lead is 1.41 times that in each of the outer wires,

$$\text{Per cent volt loss in outer wire} = \frac{10.84}{1 + 1.41} = 4.50 \text{ per cent.}$$

With a common lead one number larger in the A. W. Gauge (26 per cent larger) than each outer wire,

Approximate per cent volt loss in common lead

$$= \frac{10.84 - 4.5}{1.26} = 5.03.$$

Therefore, in the above case with No. 00 outer wires and No. 000 for the common lead,

Approximate total per cent volt loss $= 4.50 + 5.03 = 9.53$ per cent.

Amperes per wire. — From Table 17, page 52, $B = 0.500$.

From (59), in each outer wire,

$$I = \frac{0.5 \times 2000}{20 \times 0.98} = 51.0 \text{ amperes.}$$

In common wire, $I = 51 \times 1.41 = 71.9$ amperes.

Example 17. — Two substations located respectively at 40 and 50 miles away are supplied from a No. 00, 25-cycle three-phase aluminum line with wires on 60-inch centers. The total load at the nearer substation (No. 1) is 2000 kw. at 95 per cent lagging power-factor, while the other (No. 2) takes 1000 kw. at 98 per cent lagging power-factor, the generator voltage being 33,000 and the temperature 30° cent.

GIVEN ITEMS.

For No. 1: $W = 2000$ kw.; $E_0 = 33$ kv.; $L = 40$ miles; $K = 0.95$ lag; $M = 0.690\ (c - d)$.

For No. 2: $W = 1000$ kw.; $L = 10$ miles; $K = 0.98$ lag; $M = 0.655\ (c - d)$.

From Table 16, page 52, $A = 0.104$.

REQUIRED ITEMS.

Per cent volt drop to No. 1.—From (51),

$$V_0'' = \frac{0.690 \times 0.104 \times 2000 \times 40}{(33)^2} = 5.26\ (c - d).$$

From Table 15, page 51, $V_0 = 6.26$ per cent.

Kilovolts at No. 1.—From (53), $E = 33\ (1 - 0.0626) = 30.9$ kv.

Per cent volt drop from No. 1 *to No.* 2.—$E_0 = 30.9$ kv. from above.

From (51),

$$V_0'' = \frac{0.655 \times 0.104 \times 1000 \times 10}{(30.9)^2} = 0.71\ (c - d).$$

From Table 15, page 51, $V_0 = 0.81$ per cent.

Kilovolts at No. 2.—From (53), $E = 30.9\ (1 - 0.0081) = 30.6$ kv.

Example 18.* — A three-phase load of 5000 kw. and 25 cycles is to be delivered at 30,000 volts to a receiver having 95 per cent lagging power-factor, over 40 miles of No. 00 copper wires of 100 per cent conductivity on 48-inch centers, at a temperature of 20° cent.

GIVEN ITEMS.

$W = 5000$ kw.; $E = 30$ kv.; $L = 40$ miles; $K = 0.95$ lag.

By interpolation from Table 19, page 54, $M = 0.457\ (c - d)$.

From Table 16, page 52, $A = 0.100$.

REQUIRED ITEMS.

Per cent volt loss.—From (50),

$$V'' = \frac{0.457 \times 0.1 \times 5000 \times 40}{(30)^2} = 10.1\ (c - d).$$

* Problem of H. Pender, Proceedings of A.I.E.E., June, 1908, page 771.

From Table 15, page 51, $V = 11.3$ per cent. [11.3 per cent.]

Kilovolts at source. — From (52),

$$E_0 = 30\ (1 + 0.113) = 33.4 \text{ kv.} \qquad [33.4 \text{ kv.}]$$

Volt loss. — From (54),

$$v = 1000\ (33.4 - 30.0) = 3400 \text{ volts.} \qquad [3400 \text{ volts.}]$$

Per cent power loss. — From Table 3, page 7, $R = 0.411$ ohm. From (55),

$$P = \frac{0.411 \times 0.1 \times 5000 \times 40}{(30)^2 \times (0.95)^2} = 10.1 \text{ per cent.} \quad [10.1 \text{ per cent.}]$$

Kilowatts at source. — From (57),

$$W_0 = 5000\ (1 + 0.101) = 5505 \text{ kw.} \qquad [5505 \text{ kw.}]$$

Amperes per wire. — From Table 17, page 52, $B = 0.578$. From (59),

$$I = \frac{0.578 \times 5000}{30 \times 0.95} = 101.4 \text{ amperes.} \qquad [101.4 \text{ amperes.}]$$

From Table 4, page 8, No. 00 wires have ample current-carrying capacity.

Power-factor at source. — From (61),

$$K_0 = \frac{(1 + 0.101)\ 0.95}{1 + 0.113} = 0.94 \text{ lag.} \qquad [0.94 \text{ lag.}]$$

Example 19. — A three-phase load of 10,000 kw. at 60 cycles per second and 98 per cent leading power-factor is to be delivered over 65 miles of three aluminum wires on 60-inch centers, with a loss of 15 per cent of the 44,000 volts generated; temperature 40° cent.

GIVEN ITEMS.

$W = 10{,}000$ kw.; $E_0 = 44$ kv.; $V_0 = 15$ per cent;
$L = 65$ miles; $K = 0.98$ lead.

From Table 16, page 52, $A = 0.108$.
From Table 15, page 51, $V_0' = 10.9$.

REQUIRED ITEMS.

Size of each wire. — From (47),

$$M = \frac{(44)^2 \times 10.9}{0.108 \times 10{,}000 \times 65} = 0.301.$$

From Table 26, page 61, use No. 0000, for which $M = 0.316$ $(a - b)$.

Per cent volt loss. — From (51),

$$V_0'' = \frac{0.316 \times 0.108 \times 10{,}000 \times 65}{(44)^2} = 11.4\,(a - b).$$

From Table 15, page 51, $V_0 = 17.3$ per cent. [17.4 per cent.]

Per cent power loss. — From Table 3, page 7, $R = 0.417$ ohm. From (56),

$$P = \frac{0.417 \times 0.108 \times 10{,}000 \times 65}{[44\,(1 - 0.173)\,0.98]^2} = 23.0 \text{ per cent. [23.0 per cent.]}$$

Power-factor at source. — From (61),

$$K_0 = (1 + 0.23)\,(1 - 0.173)\,0.98 = 1.00. \quad \text{[0.994 lead.]}$$

Example 20.* — A three-phase load of 7500 kw. at 90 per cent lagging power-factor and 60 cycles per second is to be delivered over 140 miles of three copper wires of 100 per cent conductivity on 96-inch centers, with a voltage loss of 18.7 per cent of the volts at load, the voltage at the source being 71,200. Include the effect of capacity and assume a temperature of 20° cent.

GIVEN ITEMS.

$W = 7500$ kw.; $E_0 = 71.2$ kv.; $V = 18.7$ per cent;
$L = 140$ miles; $K = 0.90$ lag.

From Table 16, page 52, $A = 0.100$.

From (63), $V_0 = \dfrac{18.7}{1 + 0.187} = 15.8$ per cent.

From Table 15, page 51, $V_0' = 11.2$.

REQUIRED ITEMS.

Apparent power-factor of load. — Assuming that the required size of wire will be between Nos. 0000 and 8, c is approximately 0.00005. (From Table 13, page 39.)

From Table 14, page 39, the reactance factor corresponding to 90 per cent lagging power-factor = 0.49.

* Based on Problem of H. Pender, Proceedings of A.I.E.E., June, 1908, page 774.

From (44), page 38,

$$t' = 0.49 - \frac{0.00005\,[71.2\ (1 - 0.158)]^2\ 60 \times 140}{7500} = 0.29.$$

By interpolation in Table 14, page 39, the apparent power-factor of load, K', = 0.96 lag

Size of each wire. — From (47),

$$M = \frac{(71.2)^2 \times 11.2}{0.1 \times 7500 \times 140} = 0.541.$$

From Table 21, page 56, use No. 00, for which M is found by interpolation to be 0.600 $(c - d)$.

Per cent volt loss. — From (51),

$$V_0'' = \frac{0.600 \times 0.1 \times 7500 \times 140}{(71.2)^2} = 12.4\ (c - d).$$

From Table 15, Page 51, V_0 = 16.7 per cent. [17.2 per cent.]

Kilovolts at load. — From (53),

$$E = 71.2\ (1 - 0.167) = 59.3 \text{ kv.} \qquad [59.0 \text{ kv.}]$$

Per cent power loss. — From Table 3, page 7, R = 0.411 ohm. From (55),

$$P = \frac{0.411 \times 0.1 \times 7500 \times 140}{(59.3)^2\ (0.96)^2} = 13.3 \text{ per cent.} \quad [13.3 \text{ per cent.}]$$

Kilowatts at source. — From (57),

$$W_0 = 7500\ (1 + 0.133) = 8500 \text{ kw.} \qquad [8500 \text{ kw.}]$$

Power-factor at generator (not including capacity). — From (61),

$$K_0' = (1 + 0.133)\ (1 - 0.167)\ 0.96 = 0.91.$$

From Table 14, page 39, t_0' = 0.46.

From (45), page 38,

$$t_0 = 0.46 - \frac{0.00005 \times (71.2)^2 \times 60 \times 140}{8500} = 0.21.$$

From Table 14, page 39, the true power-factor at load, K_0 (corresponding to t_0), = 0.98 lag. [0.98 lag.]

Formulas for A. C. Transmission by Overhead Wires.

Required Items.	For Voltage Given at Load.	For Voltage Given at Source.
Size of each wire from per cent volt loss.	A from Table 16, page 52.	
	V' from Table 15, page 51.	V_0' from Table 15, page 51.
	$M = \frac{E^2V'}{AWL}$ . . . . (46)	$M = \frac{E_0^2V_0'}{AWL}$ . . . . (47)
	Then find wire from Tables 18 to 27, pages 53 to 62.	
Size of each wire from per cent power loss.	$R = \frac{PE^2K^2}{AWL}$ . . . (48)	$R = \frac{P[E_0(1-0.01V_0)K]^2}{AWL}$ (49)
	Then find wire from Table 3, page 7.	
Per cent volt loss.	$V'' = \frac{MAWL}{E^2}$ . . . (50)	$V_0'' = \frac{MAWL}{E_0^2}$ . . . (51)
	V from Table 15, page 51.	V_0 from Table 15, page 51.
Kilovolts at source or load.	$E_0 = E(1+0.01V)$. (52)	$E = E_0(1-0.01V_0)$. (53)
Volt loss.	$v = 1000(E_0 - E) = 10VE = 10V_0E_0$ . . . (54)	
Per cent power loss.	R from Table 3, page 7.	
	$P = \frac{RAWL}{E^2K^2}$ . . . (55)	$P = \frac{RAWL}{[E_0(1-0.01V_0)K]^2}$ (56)
Kilowatts at source.	$W_0 = W(1+0.01P)$ . . . . . (57)	
Watt loss.	$p = 1000(W_0 - W) = 10PW$. . (58)	
Amperes per wire. (See par. 34, page 35.)	B from Table 17, page 52.	
	$I = \frac{BW}{EK}$ . . . . (59)	$I = \frac{BW}{E_0(1-0.01V_0)K}$. (60)
Power-factor at source.	$K_0 = \frac{(1+0.01P)K}{1+0.01V} = (1+0.01P)(1-0.01V_0)K$. (61)	

When given E and V_0, find V from $V = V_0/(1-0.01V_0)$ . . . . . (62)
When given E_0 and V, find V_0 from $V_0 = V/(1+0.01V)$ . . . . . (63)
When given W', find W from $W = W'K$ . . . . . (64)

NOTATION.

A = Transmission factor. Table 16, page 52.
B = Transmission constant. Table 17, p. 52.
E = Kilovolts (1000 volts) between wires at load.
E_0 = Kilovolts (1000 volts) between wires at source. See par. 32, p. 35.
I = Amperes per wire. See par. 32, p. 35.
K = Power-factor of load, in decimals.
K_0 = Power-factor at source, in decimals.
L = Distance from source to load, in miles or 1000 feet.
M = Wire factor Tables 18 to 27, pp. 53 to 62.
P = Power loss in per cent of power at load.
p = Power loss in watts.
R = Resistance per mile of one wire, in ohms. Table 3, p. 7.
V = Volt loss in per cent of volts at load.
V_0 = Volt loss in per cent of volts at source.
V', V_0', V'', V_0'' = Volt loss factors. Table 15, p. 51
v = Volt loss (volts at source — volts at load).
W = Real kilowatts at load.
W_0 = Real kilowatts at source.
W' = Kilovolt-amperes (apparent power) at load.

Table 15. — Values of Volt Loss Factors.

V or V_0	V'	V''			V_0'	V_0''			V or V_0	V'	V''			V_0'	V_0''		
		a-b	*b-c*	*c-d*		*a-b*	*b-c*	*c-d*			*a-b*	*b-c*	*c-d*		*a-b*	*b-c*	*c-d*
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	8.0	7.4	7.5	7.4	7.2	6.8	6.8	6.8	6.6
0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	8.2	7.6	7.7	7.6	7.3	6.9	7.0	6.9	6.8
0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	8.4	7.8	7.9	7.8	7.5	7.0	7.1	7.1	6.9
0.6	0.6	0.6	0.6	0.5	0.6	0.6	0.6	0.6	8.6	7.9	8.0	8.0	7.7	7.2	7.3	7.2	7.1
0.8	0.8	0.8	0.8	0.7	0.8	0.8	0.8	0.7	8.8	8.1	8.2	8.1	7.9	7.3	7.4	7.4	7.2
1.0	1.0	1.0	1.0	0.9	1.0	1.0	1.0	0.9	9.0	8.3	8.4	8.3	8.0	7.5	7.5	7.5	7.4
1.2	1.2	1.2	1.2	1.1	1.2	1.2	1.2	1.1	9.2	8.4	8.5	8.5	8.2	7.6	7.6	7.6	7.5
1.4	1.4	1.4	1.4	1.3	1.4	1.4	1.3	1.2	9.4	8.6	8.7	8.6	8.4	7.7	7.8	7.8	7.7
1.6	1.6	1.6	1.5	1.5	1.6	1.6	1.5	1.4	9.6	8.8	8.9	8.8	8.5	7.8	7.9	7.9	7.8
1.8	1.8	1.8	1.7	1.7	1.7	1.8	1.7	1.6	9.8	8.9	9.0	9.0	8.7	8.0	8.0	8.1	7.9
2.0	2.0	2.0	1.9	1.8	1.9	2.0	1.9	1.8	10.0	9.1	9.2	9.1	8.9	8.1	8.1	8.2	8.1
2.2	2.2	2.2	2.1	2.0	2.1	2.2	2.1	1.9	10.2	9.2	9.3	9.3	9.1	8.2	8.2	8.3	8.2
2.4	2.3	2.4	2.3	2.2	2.3	2.3	2.2	2.1	10.4	9.4	9.5	9.5	9.3	8.4	8.4	8.5	8.4
2.6	2.5	2.6	2.5	2.4	2.5	2.5	2.4	2.2	10.6	9.6	9.7	9.7	9.5	8.5	8.5	8.6	8.5
2.8	2.7	2.8	2.7	2.6	2.6	2.7	2.6	2.4	10.8	9.7	9.8	9.8	9.6	8.6	8.6	8.8	8.7
3.0	2.9	3.0	2.8	2.7	2.8	2.9	2.8	2.6	11.0	9.9	10.0	10.0	9.8	8.7	8.7	8.9	8.9
3.2	3.1	3.2	3.0	2.9	3.0	3.1	3.0	2.8	11.2	10.1	10.1	10.1	10.0	8.9	8.8	9.0	9.0
3.4	3.3	3.4	3.2	3.1	3.2	3.2	3.1	2.9	11.4	10.2	10.3	10.3	10.2	9.0	8.9	9.1	9.1
3.6	3.5	3.6	3.4	3.3	3.3	3.4	3.3	3.1	11.6	10.4	10.4	10.4	10.3	9.1	9.0	9.3	9.3
3.8	3.7	3.8	3.6	3.5	3.5	3.6	3.5	3.3	12.0	10.7	10.8	10.8	10.7	9.3	9.2	9.5	9.5
4.0	3.8	4.0	3.8	3.6	3.7	3.8	3.6	3.4	12.5	11.1	11.1	11.3	11.1	9.6	9.5	9.8	9.8
4.2	4.0	4.2	4.0	3.8	3.8	3.9	3.8	3.6	13.0	11.5	11.5	11.7	11.6	9.8	9.7	10.1	10.2
4.4	4.2	4.4	4.2	4.0	4.0	4.1	4.0	3.8	13.5	11.9	11.9	12.1	12.0	10.1	9.9	10.4	10.5
4.6	4.4	4.5	4.3	4.2	4.2	4.3	4.1	3.9	14.0	12.3	12.2	12.6	12.5	10.4	10.2	10.7	10.8
4.8	4.6	4.7	4.5	4.4	4.3	4.4	4.3	4.1	14.5	12.7	12.6	13.0	12.9	10.6	10.4	11.0	11.1
5.0	4.8	4.9	4.7	4.5	4.5	4.6	4.4	4.3	15.0	13.1	13.0	13.4	13.4	10.9	10.6	11.2	11.4
5.2	4.9	5.1	4.9	4.7	4.7	4.8	4.6	4.4	15.5	13.4	13.3	13.8	13.8	11.1	10.8	11.5	11.7
5.4	5.1	5.3	5.1	4.9	4.8	4.9	4.8	4.6	16.0	13.8	13.6	14.2	14.2	11.3	11.0	11.8	12.0
5.6	5.3	5.5	5.3	5.1	5.0	5.1	4.9	4.7	16.5	14.2	14.0	14.6	14.7	11.5	11.2	12.0	12.3
5.8	5.5	5.7	5.5	5.2	5.1	5.3	5.1	4.9	17.0	14.5	14.3	15.0	15.1	11.7	11.3	12.3	12.6
6.0	5.7	5.8	5.6	5.4	5.3	5.4	5.3	5.1	17.5	14.9	14.7	15.4	15.6	11.9	11.5	12.5	12.9
6.2	5.8	6.0	5.8	5.6	5.4	5.6	5.4	5.2	18.0	15.3	15.0	15.8	16.0	12.1	11.6	12.7	13.2
6.4	6.0	6.2	6.0	5.8	5.6	5.7	5.6	5.4	18.5	15.6	15.3	16.2	16.5	12.3	11.8	13.0	13.4
6.6	6.2	6.4	6.2	5.9	5.8	5.9	5.7	5.5	19.0	16.0	15.6	16.6	16.9	12.5	11.9	13.2	13.7
6.8	6.4	6.5	6.3	6.1	5.9	6.0	5.9	5.7	20.0	16.7	16.2	17.4	17.8	12.8	12.2	13.6	14.2
7.0	6.5	6.7	6.5	6.3	6.1	6.2	6.0	5.8	21.0	17.4	16.8	18.2	18.7	....	....	....	....
7.2	6.7	6.9	6.7	6.5	6.2	6.3	6.2	6.0	22.0	18.0	17.4	19.0	19.6	....	....	....	....
7.4	6.9	7.0	6.9	6.6	6.3	6.4	6.3	6.2	23.0	18.7	18.0	19.7	20.5	....	....	....	....
7.6	7.1	7.2	7.1	6.8	6.5	6.6	6.5	6.3	24.0	19.4	18.5	20.5	21.4	....	....	....	....
7.8	7.2	7.4	7.3	7.0	6.6	6.7	6.6	6.5	25.0	20.0	19.1	21.3	22.3	....	....	....	....

Table 16 — Values of *A* for Balanced Loads.

Temperature in degrees.		1-Phase System with 2 Equal Wires		2-Phase System with 3 Equal Wires.		2-Phase System with 4 Equal Wires, or 3-Phase System with 3 Equal Wires.	
Cent.	Fahr	For *L* in Miles.	For *L* in 1000 Ft.	For *L* in Miles.	For *L* in 1000 Ft.	For *L* in Miles.	For *L* in 1000 Ft.
0	32	0.185	0.0351	0.112	0.0212	0.0926	0.0175
10	50	0.193	0.0365	0.116	0.0220	0.0963	0.0182
20	68	0.200	0.0379	0.121	0 0229	0.100	0.0190
30	86	0.208	0.0393	0.125	0.0237	0.104	0.0197
40	104	0.215	0.0408	0.130	0.0246	0 108	0.0204
50	122	0 223	0.0423	0.135	0.0255	0.112	0.0212

The above values are for copper wires of 100 per cent conductivity and aluminum wires of 62 per cent conductivity, in Matthiessen's standard scale.

For copper of other conductivity, divide *A* by given conductivity

For aluminum of other conductivity, divide *A* by given conductivity and multiply by 0.62

Table 17. — Values of *B* for Balanced Loads.

1-Phase System.	2-Phase System.	3-Phase System.
1.000	0.500	0.578

Table 18. — Values of *M* for Overhead Copper Wires at 15 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
500,000	[a] .075	[b] .087	[b] .105	[e] .124	[c] .138	[e] .154	[e] .169	[e] .181	[e] .197	[a] .070	[a] .085	.107	[e] .129	[e] .145	[e] .165	[e] .181	[e] .199	[e] .213	[a] .068	[a] .084	[b] .109	[e] .133	[a] .149	[e] .169	.187	[a] .204	[a] .224
400,000	.099	.111	[c] .129	.148	.162	.179	.194	.207	.224	.094	[b] .109	.131	.153	.169	.190	.206	.225	.240	.092	.108	.133	.157	.173	.194	.212	.230	.251
300,000	[b] .137	.150	.168	.188	.203	.221	.237	.251	.269	[b] .132	.148	[a] .170	.193	.210	.232	.249	.268	.294	.131	[b] .147	[a] .172	.197	.214	.236	.255	.275	.296
250,000	.169	[a] .182	.200	.223	.238	.256	.272	.287	.305	.164	.180	.203	.228	.245	.267	.284	.303	.330	[b] .163	.179	.205	.231	.248	.271	.290	.311	.332
0000	.203	.217	.235	.258	.274	.294	.310	.326	.345	.199	[a] .215	.238	.253	.280	.303	.322	.342	.370	.198	[a] .219	.240	.266	.284	.309	.328	.350	.372
000	[c] .263	.276	.296	.319	.336	.356	.372	.388	.408	[a] .259	.275	.298	.314	.342	.365	.385	.405	.434	[a] .257	.273	.300	.327	.346	.371	.390	.413	.436
00	.337	.350	.370	.394	.412	.432	.450	.467	.488	.333	.349	.373	.341	.419	.443	.464	.484	.515	.330	.346	.374	.403	.424	.450	.470	.494	.518
0	.435	.448	.468	.493	.513	.534	.552	.570	.591	.428	.444	.468	.487	.515	.539	.562	.582	.617	.425	.441	.469	.500	.523	.549	.569	.593	.620
1	.555	.568	.588	.614	.636	.659	.678	.696	.719	.547	.563	.588	.610	.640	.664	.688	.709	.745	.543	.559	.587	.620	.644	.673	.693	.720	.748
2	.708	.721	.741	.768	.790	.815	.834	.854	.881	.700	.716	.741	.764	.794	.820	.844	.867	.908	.696	.712	.741	.774	.799	.829	.849	.878	.910
3	.904	.917	.938	.966	.988	1.01	1.03	1.05	1.08	.893	.910	.935	.956	.987	1.02	1.04	1.06	1.10	.890	.906	.935	.970	.997	1.03	1.05	1.08	1.11
4	1.14	1.15	1.18	1.21	1.23	1.26	1.28	1.31	1.34	1.14	1.15	1.18	1.20	1.23	1.27	1.29	1.31	1.36	1.14	1.15	1.18	1.22	1.25	1.28	1.30	1.33	1.37
5	1.45	1.46	1.49	1.52	1.54	1.57	1.59	1.62	1.66	1.45	1.46	1.49	1.51	1.54	1.58	1.60	1.62	1.68	1.45	1.46	1.49	1.53	1.56	1.59	1.61	1.65	1.69
6	1.84	1.85	1.88	1.91	1.94	1.97	1.99	2.03	2.07	1.84	1.85	1.88	1.90	1.94	1.98	2.00	2.02	2.09	1.84	1.85	1.88	1.92	1.95	1.98	2.00	2.05	2.10
7	2.33	2.34	2.37	2.40	2.43	2.46	2.48	2.53	2.58	2.33	2.34	2.37	2.39	2.43	2.47	2.49	2.52	2.60	2.32	2.33	2.36	2.40	2.44	2.47	2.49	2.55	2.61
8	2.94	2.95	2.98	3.02	3.05	3.08	3.11	3.16	3.22	2.94	2.95	2.98	3.01	3.05	3.09	3.12	3.15	3.24	2.94	2.95	2.98	8.02	3.06	3.09	3.11	3.18	3.25
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 19. — Values of *M* for Overhead Copper Wires at 25 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
500,000	....	[a] .082	[a] .115	[b] .145	[c] .169	[c] .195	[c] .218	[c] .244	[c] .253	....	....	[a] .117	[b] .156	[b] .181	[c] .211	[c] .238	[c] .265	[c] .292	....	....	[a] .120	[b] .160	[b] .189	[c] .221	[c] .249	[c] .275	[c] .310
400,000	....	106	[b] .139	.169	.193	.220	.244	.270	.281	....	[a] .103	.141	.180	[c] .205	.236	.264	.292	.320	....	....	.144	.184	[c] .213	.247	.276	.303	.339
300,000	[a] .122	.144	.177	[c] .208	.232	.260	.286	.313	.326	[a] .117	.141	[b] .179	[c] .219	.244	.277	.306	.336	.365	....	[a] .137	[b] .182	[c] .223	.252	.289	.319	.348	.386
250,000	.153	[b] .175	.208	.240	.264	.294	.321	.349	.364	.148	.173	.213	.252	.278	.312	.342	.373	.403	[a] .144	.169	.215	.256	.285	.324	.355	.385	.425
0000	[b] .187	.210	[c] .243	.275	.299	.330	.359	.387	.406	.182	[b] .207	.247	.287	.313	.349	.380	.413	.445	.177	[b] .203	.250	.291	.320	.360	.393	.425	.467
000	.247	.270	.304	.339	.363	.394	.424	.453	.474	[b] .239	.264	[c] .305	.347	.375	.413	.445	.479	.513	.236	.262	[c] .309	.352	.382	.424	.457	.491	.535
00	.319	[c] .342	.377	.416	.442	.474	.505	.534	.556	.311	.337	.380	.423	.453	.492	.524	.558	.594	[b] .309	.335	.383	.429	.462	.503	.536	.571	.617
0	[c] .413	.436	.473	.517	.543	.575	.608	.637	.660	.404	[c] .430	.474	.518	.550	.590	.622	.658	.697	.403	[c] .429	.478	.526	.560	.601	.637	.675	.721
1	.532	.555	.593	.641	.667	.699	.732	.761	.792	[c] .523	.549	.596	.641	.673	.813	.746	.785	.831	[c] .522	.548	.600	.650	.685	.728	.764	.802	.855
2	.683	.706	.744	.792	.819	.854	.890	.919	.953	.675	.701	.749	.795	.828	.869	.902	.942	.990	.671	.697	.749	.802	.838	.882	.918	.957	1.01
3	.867	.895	.933	.986	1.01	1.05	1.09	1.12	1.16	.867	.893	.941	.990	1.03	1.07	1.10	1.14	1.19	.859	.885	.937	.990	1.03	1.07	1.11	1.15	1.21
4	1.11	1.14	1.18	1.23	1.25	1.30	1.34	1.37	1.41	1.10	1.13	1.18	1.23	1.27	1.32	1.35	1.39	1.44	1.10	1.13	1.18	1.23	1.28	1.32	1.36	1.40	1.47
5	1.42	1.45	1.49	1.54	1.57	1.62	1.66	1.69	1.73	1.41	1.44	1.49	1.54	1.59	1.64	1.67	1.71	1.76	1.41	1.44	1.49	1.54	1.60	1.64	1.68	1.72	1.79
6	1.81	1.84	1.88	1.93	1.96	2.01	2.05	2.09	2.14	1.80	1.83	1.88	1.93	1.98	2.03	2.06	2.11	2.17	1.80	1.83	1.88	1.93	1.99	2.03	2.08	2.13	2.20
7	2.29	2.32	2.36	2.41	2.44	2.49	2.54	2.59	2.65	2.28	2.31	2.36	2.41	2.46	2.52	2.55	2.61	2.68	2.28	2.31	2.36	2.41	2.47	2.52	2.57	2.63	2.71
8	2.91	2.94	2.98	3.03	3.07	3.13	3.18	3.24	3.30	2.90	2.93	2.98	3.03	3.08	3.15	3.20	3.26	3.33	2.90	2.93	2.98	3.03	3.09	3.16	3.22	3.28	3.36
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 20. — Values of *M* for Overhead Copper Wires at 40 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
500,000	....	....	[a] .130	[a] .183	[b] .215	[b] .256	[c] .291	[c] .328	[c] .365	....	....	....	[a] .198	[b] .236	[b] .286	[c] .326	[c] .368	[c] .411	....	....	....	[a] .208	[b] .248	[b] .304	[c] .347	[c] .393	[c] .439
400,000	....	....	.155	[b] .208	[c] .241	[c] .283	.319	.356	.394	....	....	[a] .157	[b] .223	.252	.313	.354	.396	.441	....	....	[a] .161	.233	.274	.331	.375	.421	.469
300,000	....	[a] .135	.195	.249	[c] .282	.325	.362	.401	.439	....	....	.197	.264	.293	[c] .355	.397	.441	.487	....	....	.201	[b] .274	.315	[c] .373	.418	.476	.515
250,000	....	.166	[b] .226	.281	.315	.359	.398	.438	.478	....	[a] .162	.228	.296	[c] .326	.389	.432	.478	.526	....	....	.232	.306	.348	.407	.453	.513	.554
0000	[a] .165	.199	.260	[c] .316	.350	.396	.437	.479	.520	....	.195	[b] .262	.330	.361	.426	.469	.518	.568	....	[a] .194	.264	.339	[c] .383	.444	.492	.554	.596
000	.224	.259	.320	.376	.412	.461	.506	.550	.593	[a] .209	.254	.321	[c] .390	.423	.491	.536	.590	.641	[a] .203	.253	[b] .323	[c] .399	.445	.507	.558	.624	.668
00	.296	[b] .333	[c] .394	.451	.487	.538	.585	.632	.677	.282	[b] .329	.396	.466	.500	.569	.616	.672	.735	.275	.326	.396	.474	.523	.586	.638	.707	.753
0	[b] .389	.426	.487	.548	.585	.639	.689	.737	.788	[b] .373	.420	[c] .488	.559	.594	.667	.719	.778	.845	.368	[b] .420	[c] .490	.568	.620	.686	.742	.813	.864
1	.503	[c] .541	.605	.671	.710	.765	.815	.853	.915	.490	.539	.608	.683	.719	.793	.847	.907	.975	[b] .485	.537	.607	.688	.742	.812	.869	.940	.994
2	.650	.688	.754	.825	.864	.921	.972	1.01	1.07	.636	[c] .687	.760	.834	.872	.949	1.01	1.07	1.14	.633	[c] .685	.756	.841	.898	.972	1.03	1.10	1.16
3	[c] .842	.878	.949	1.02	1.06	1.12	1.17	1.21	1.27	[c] .826	.877	.953	1.03	1.07	1.15	1.21	1.27	1.34	[c] .820	.872	.943	1.04	1.10	1.16	1.23	1.30	1.36
4	1.08	1.12	1.19	1.27	1.31	1.37	1.42	1.47	1.53	1.06	1.11	1.19	1.27	1.31	1.39	1.45	1.52	1.60	1.06	1.11	1.19	1.29	1.35	1.41	1.48	1.55	1.62
5	1.38	1.42	1.49	1.57	1.62	1.69	1.74	1.79	1.86	1.36	1.41	1.49	1.57	1.62	1.70	1.76	1.83	1.92	1.36	1.41	1.49	1.59	1.66	1.72	1.79	1.86	1.95
6	1.77	1.81	1.88	1.96	2.02	2.09	2.14	2.20	2.27	1.75	1.80	1.88	1.97	2.02	2.10	2.16	2.23	2.33	1.75	1.80	1.88	1.98	2.05	2.12	2.19	2.26	2.36
7	2.25	2.29	2.36	2.44	2.51	2.58	2.63	2.70	2.79	2.24	2.29	2.37	2.46	2.52	2.60	2.66	2.74	2.85	2.24	2.29	2.37	2.47	2.54	2.62	2.69	2.77	2.88
8	2.87	2.91	2.98	3.06	3.13	3.20	3.25	3.33	3.44	2.85	2.90	2.98	3.08	3.14	3.22	3.28	3.37	3.49	2.85	2.90	2.98	3.08	3.16	3.24	3.32	3.40	3.53
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 21. — Values of *M* for Overhead Copper Wires at 60 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
500,000	....	....	....	[a] .227	[b] .273	[b] .340	[b] .391	[c] .448	[c] .506	....	....	....	[a] .252	[a] .306	[b] .383	[b] .445	[c] .511	[c] .572	....	....	....	[a] .266	[a] .328	[b] .410	[b] .477	[c] .548	[c] .620
400,000	....	....	....	.252	.299	.368	[c] .420	.478	.538	....	....	....	.277	[b] .332	.411	.474	.541	.604	....	....	....	.291	[b] .354	.438	.507	.579	.652
300,000	....	....	[a] .210	[b] .293	.342	[c] .412	.465	.524	.585	....	....	[a] .219	.318	.375	.455	[c] .519	.587	.651	....	....	....	.332	.397	.482	[c] .552	.625	.699
250,000	....	....	.242	.328	.379	.450	.504	.563	.625	....	....	.250	[b] .352	.412	[c] .494	.558	.626	.692	....	....	[a] .256	.366	.432	.518	.590	.664	.740
0000	....	....	.276	.363	.416	.487	.545	.604	.667	....	....	.284	.388	.451	.534	.599	.668	.736	....	....	.291	[b] .403	.471	[c] .557	.630	.705	.783
000	....	[a] .245	[b] .336	.424	[c] .479	.551	.613	.676	.743	....	....	.345	.450	.516	.604	.673	.745	.815	....	....	.352	.465	.535	.623	.700	.778	.858
00	[a] .262	.318	.409	[c] .498	.564	.630	.695	.763	.833	....	[a] .312	[b] .419	.526	[c] .594	.686	.759	.832	.906	....	[a] .308	.425	.538	[c] .611	.707	.787	.866	.949
0	.352	.409	.503	.593	.663	.736	.808	.878	.948	[a] .337	.404	.512	[c] .619	.695	.792	.868	.943	1.02	[a] .326	.399	[b] .517	[c] .632	.706	.803	.886	.972	1.06
1	.470	[b] .529	[c] .623	.714	.784	.860	.936	1.01	1.08	.454	[b] .522	.631	.740	.816	.915	.993	1.07	1.15	.443	.516	.634	.750	.824	.925	1.01	1.10	1.19
2	[b] .615	.675	.771	.865	.937	1.02	1.10	1.17	1.24	[b] .600	.668	[c] .777	.889	.966	1.06	1.15	1.23	1.31	.590	[b] .663	[c] .781	.898	.972	1.08	1.17	1.26	1.35
3	.800	[c] .862	.960	1.06	1.13	1.22	1.30	1.37	1.44	.791	.859	.968	1.08	1.16	1.26	1.35	1.43	1.52	[b] .779	.852	.971	1.09	1.16	1.27	1.37	1.46	1.55
4	[c] 1.03	1.09	1.19	1.31	1.38	1.47	1.55	1.63	1.70	1.02	[c] 1.09	1.20	1.32	1.41	1.51	1.60	1.69	1.78	1.01	[c] 1.09	1.22	1.34	1.41	1.52	1.62	1.71	1.81
5	1.34	1.40	1.50	1.62	1.69	1.79	1.87	1.95	2.03	[c] 1.33	1.40	1.51	1.63	1.72	1.82	1.92	2.01	2.11	[c] 1.31	1.39	1.52	1.65	1.73	1.84	1.95	2.04	2.14
6	1.72	1.78	1.88	2.01	2.08	2.19	2.27	2.35	2.44	1.72	1.79	1.90	2.02	2.12	2.22	2.32	2.41	2.52	1.69	1.77	1.91	2.04	2.13	2.24	2.35	2.44	2.55
7	2.20	2.26	2.36	2.49	2.57	2.69	2.77	2.85	2.96	2.19	2.26	2.37	2.49	2.60	2.71	2.81	2.90	3.03	2.16	2.24	2.38	2.52	2.61	2.72	2.83	2.94	3.07
8	2.82	2.88	2.98	3.12	3.20	3.32	3.41	3.50	3.61	2.81	2.88	2.99	3.12	3.23	3.34	3.45	3.54	3.67	2.78	2.86	3.00	3.15	3.24	3.35	3.47	3.58	3.72
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 22. — Values of *M* for Overhead Copper Wires at 125 Cycles per Second.

B. & S. or A.W.G.	Wires on 18-Inch Centers.							Wires on 36-Inch Centers.							Wires on 60-Inch Centers.						
	Power-Factor of Load in Per Cent.																				
	Lag.							Lag.							Lag.						
	100	98	95	90	85	80	75	100	98	95	90	85	80	75	100	98	95	90	85	80	75
0000	.....	[a] 0.528	[b] 0.639	[b] 0.787	[b] 0.907	[c] 1.03	[c] 1.15	.....	[a] 0.574	[a] 0.706	[b] 0.884	[b] 1.03	[c] 1.17	[c] 1.32	.....	[a] 0.609	[a] 0.750	[b] 0.935	[b] 1.09	[c] 1.25	[c] 1.39
000	.....	0.597	0.714	0.868	[c] 0.992	1.12	1.24	.....	0.639	[b] 0.776	0.964	[c] 1.11	1.25	1.40	.....	0.672	[b] 0.817	1.01	1.17	1.33	1.48
00	[a] 0.481	0.690	0.810	0.964	1.09	1.22	1.35	[a] 0.497	0.730	0.872	1.06	1.21	1.35	1.51	.....	0.762	0.912	1.11	[c] 1.27	1.44	1.59
0	0.574	[b] 0.784	0.905	[c] 1.06	1.19	1.32	1.46	0.591	[b] 0.824	0.970	[c] 1.16	1.31	1.46	1.62	[a] 0.606	0.860	1.01	1.21	1.37	1.54	1.70
1	0.688	0.898	[c] 1.02	1.18	1.32	1.45	1.60	0.715	0.948	1 09	1.28	1.44	1.59	1.75	0.723	[b] 0.980	1.13	[c] 1.33	1.50	1.67	1.84
2	[b] 0.840	1.05	1.17	1.34	1.48	1.62	1.77	0.857	1.09	[c] 1.24	1.44	1.60	1.75	1.92	0.870	1.13	[c] 1.28	1.49	1.66	1.83	2.01
3	1.03	[c] 1.24	1.36	1.54	1.69	1.84	1.99	[b] 1.04	[c] 1.27	1.42	1.63	1.80	1.96	2.13	[b] 1.04	1.30	1.46	1.68	1.86	2.04	2.23
4	1.26	1.47	1.60	1.79	1.94	2.10	2.26	1.27	1.50	1.66	1.88	2.05	2.22	2.40	1.27	[c] 1.53	1.70	1.93	2.11	2.30	2.50
5	[c] 1.56	1.77	1.91	2.11	2.26	2.43	2.60	[c] 1.57	1.80	1.97	2.20	2.37	2.55	2.74	1.57	1.84	2.01	2.25	2.44	2.63	2.83
6	1.92	2.14	2.29	2.50	2.66	2.83	3.01	1.93	2.17	2.35	2.59	2.77	2.96	3.15	[c] 1.93	2.21	2.39	2.64	2.84	3.04	3.24
7	2.42	2.65	2.80	3.02	3.18	3.35	3.54	2.42	2.67	2.86	3.10	3.29	3.48	3.67	2.42	2.71	2.89	3.15	3.36	3.56	3.76
8	3.03	3.26	3.41	3.64	3.81	3.99	4.18	3.03	3.29	3.48	3.72	3.92	4.11	4.31	3.03	3.32	3.51	3.77	3.98	4.19	4.39
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 23. — Values of *M* for Overhead Aluminum Wires at 15 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
1,000,000	[a] .058	[a] .069	[b] .037	[c] .104	[c] .116	[c] .131	[c] .144	[c] .157	[c] .170	....	[a] .068	[b] .088	[b] .109	[c] .123	[c] .140	[c] .155	[c] .169	[c] .183	....	[a] .068	[b] .090	[b] .113	[c] .129	[c] .148	[c] .164	[c] .180	[c] .194
800,000	.077	[b] .088	.106	.124	.136	.151	.165	.179	.192	[a] .073	.087	.107	[c] .128	.142	.160	.176	.191	.205	[a] .070	.086	.108	[c] .132	.148	.168	.185	.202	.216
700,000	.091	.102	[c] .120	.138	.150	.165	.179	.194	.207	.087	[b] .101	.121	.142	.156	.175	.191	.207	.221	.084	.100	.122	.146	.163	.183	.200	.218	.232
600,000	[b] .109	.120	.138	.157	.170	.180	.200	.215	.229	.105	.119	[c] .139	.161	.175	.195	.212	.228	.243	.102	[b] .118	.140	.165	.182	.203	.220	.239	.254
500,000	.134	.145	.163	.183	.196	.213	.228	.244	.258	[b] .130	.144	.165	.187	.201	.221	.239	.256	.272	[b] .127	.143	[c] .166	.191	.208	.229	.247	.268	.283
400,000	.172	[c] .183	.202	.223	.236	.254	.269	.286	.300	.167	.182	.203	.227	.242	.262	.280	.298	.314	.165	.181	.205	.231	.248	.269	.287	.308	.323
300,000	[c] .236	.247	.267	.288	.302	.321	.336	.354	.368	.230	[c] .245	.268	.293	.309	.331	.349	.368	.385	.229	[c] .245	.269	.296	.314	.336	.356	.377	.392
250,000	.289	.300	.320	.342	.354	.376	.391	.410	.425	[c] .282	.298	.321	.346	.363	.386	.404	.423	.442	[c] .282	.298	.322	.350	.368	.391	.412	.433	.448
0000	.343	.354	.374	.399	.414	.433	.449	.469	.486	.336	.352	.375	.402	.420	.444	.463	.483	.503	.336	.352	.377	.407	.425	.449	.470	.491	.508
000	.441	.452	.473	.499	.515	.536	.553	.573	.592	.433	.449	.473	.501	.520	.547	.566	.587	.608	.433	.449	.475	.506	.525	.552	.573	.595	.614
00	.566	.577	.598	.624	.640	.662	.680	.702	.723	.556	.572	.597	.626	.645	.673	.692	.716	.739	.550	.572	.598	.630	.650	.677	.700	.724	.746
0	.720	.731	.752	.778	.794	.819	.838	.861	.887	.710	.721	.753	.782	.802	.831	.850	.876	.903	.710	.726	.752	.785	.806	.836	.861	.886	.911
1	.914	.926	.947	.973	.990	1.02	1.04	1.06	1.09	.905	.920	.949	.979	1.00	1.03	1.05	1.07	1.10	.904	.920	.946	.979	1.00	1.03	1.05	1.08	1.11
2	1.17	1.18	1.20	1.23	1.25	1.28	1.30	1.32	1.36	1.16	1.17	1.20	1.23	1.25	1.28	1.31	1.33	1.37	1.16	1.17	1.20	1.23	1.25	1.29	1.31	1.34	1.38
3	1.48	1.49	1.51	1.54	1.56	1.59	1.61	1.64	1.68	1.47	1.48	1.51	1.54	1.56	1.59	1.62	1.64	1.69	1.47	1.48	1.51	1.54	1.56	1.60	1.62	1.65	1.70
4	1.87	1.88	1.90	1.93	1.95	1.98	2.01	2.04	2.09	1.86	1.87	1.90	1.93	1.96	1.99	2.02	2.05	2.10	1.86	1.87	1.90	1.93	1.96	2.00	2.02	2.05	2.11
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 24. — Values of *M* for Overhead Aluminum Wires at 25 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.		Lag.							Lead.		Lag.							Lead.		Lag.						
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
1,000,000	....	....	[a] .095	[b] .125	[b] .144	[c] .167	[c] .189	[c] .210	[c] .231	....	....	[a] .098	[b] .135	[b] .150	[c] .183	[c] .208	[c] .233	[c] .260	....	....	[a] .101	[b] .142	[b] .166	[c] .197	[c] .225	[c] .251	[c] .282
800,000	....	[a] .083	.114	.144	[c] .164	.189	.211	.233	.254	....	....	.117	.154	.176	.205	.230	.256	.283	....	....	.120	.161	.186	.218	.246	.273	.304
700,000	....	.096	[b] .127	.157	.177	.203	.225	.247	.269	....	[a] .094	.131	.168	[c] .190	.219	.244	.270	.298	....	....	.134	.175	.200	.232	.260	.287	.319
600,000	[a] .094	.115	.146	[a] .176	.197	.224	.247	.269	.291	....	.112	[b] .150	.187	.209	.239	.264	.290	.319	....	[a] .109	.152	.193	[c] .219	.252	.281	.308	.340
500,000	.119	.140	.171	.201	.222	.250	.274	.298	.321	[a] .113	.137	.175	[c] .212	.235	.265	.291	.318	.348	....	.134	[b] .177	[c] .218	.244	.277	.307	.335	.367
400,000	.157	[b] .178	[c] .209	.240	.261	.290	.315	.340	.365	.151	[b] .175	.213	.250	.274	.306	.333	.362	.394	[a] .148	.171	.214	.256	.283	.318	.349	.379	.412
300,000	[b] .220	.241	.273	.306	.328	.359	.385	.411	.436	[b] .214	.238	[c] .276	.315	.339	.374	.403	.434	.466	.211	[b] .234	[c] .277	.320	.347	.383	.414	.445	.480
250,000	.272	[c] .293	.325	.360	.384	.415	.441	.468	.493	.265	.289	.328	.368	.393	.428	.458	.490	.523	[b] .262	.286	.329	.373	.401	.437	.468	.500	.536
0000	.325	.346	.379	.416	.441	.474	.501	.528	.554	.318	[c] .342	.382	.425	.452	.488	.518	.551	.586	.317	.341	.384	.429	.458	.495	.529	.561	.599
000	[c] .421	.442	.476	.515	.541	.575	.602	.631	.660	.413	.437	.478	.524	.552	.590	.622	.655	.692	.411	[c] .435	.479	.530	.563	.600	.634	.667	.705
00	.543	.564	.598	.640	.677	.702	.730	.760	.793	[c] .535	.559	.601	.648	.676	.715	.747	.782	.824	[c] .532	.557	.603	.655	.690	.729	.765	.798	.836
0	.696	.717	.752	.795	.832	.863	.893	.927	.961	.688	.712	.755	.802	.833	.873	.910	.947	.997	.684	.710	.757	.809	.845	.887	.927	.961	1.00
1	.885	.906	.945	.988	1.03	1.06	1.09	1.13	1.17	.882	.906	.951	1.00	1.03	1.07	1.11	1.15	1.20	.876	.903	.951	1.00	1.04	1.08	1.13	1.16	1.21
2	1.14	1.16	1.20	1.25	1.29	1.32	1.35	1.39	1.43	1.14	1.16	1.20	1.25	1.29	1.33	1.37	1.41	1.46	1.13	1.16	1.20	1.26	1.30	1.34	1.39	1.42	1.47
3	1.45	1.47	1.51	1.56	1.60	1.63	1.66	1.70	1.74	1.45	1.47	1.51	1.56	1.60	1.64	1.68	1.72	1.78	1.44	1.47	1.51	1.57	1.61	1.65	1.70	1.73	1.79
4	1.84	1.86	1.90	1.95	1.99	2.03	2.07	2.11	2.15	1.84	1.86	1.90	1.95	1.99	2.04	2.08	2.13	2.19	1.83	1.86	1.90	1.97	2.01	2.05	2.10	2.14	2.20
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 25. — Values of *M* for Overhead Aluminum Wires at 40 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.		Lag.							Lead.		Lag.							Lead.		Lag.						
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
1,000,000			[a] .107	[a] .158	[b] .186	[b] .226	[c] .258	[c] .292	[c] .328				[a] .172	[b] .206	[b] .253	[c] .291	[c] .329	[c] .369				[a] .183	[b] .221	[b] .275	[b] .316	[c] .361	[c] .401
800,000			.127	[b] .178	.207	[c] .248	.281	.315	.352			[a] .132	.192	.228	.276	.314	.362	.393				.203	.242	.298	[c] .340	.385	.426
700,000			.140	.192	.221	.262	.295	.329	.368			.145	[b] .206	.242	.290	.328	.377	.410			[a] .150	.217	.256	.312	.354	.399	.442
600,000			.159	.212	.242	.283	.318	.353	.393			.165	.227	.263	[c] .311	.350	.400	.436			.169	[b] .237	.277	[c] .334	.376	.422	.467
500,000		[a] .131	.183	.236	[c] .267	.310	.345	.380	.420			.189	.252	.289	.337	.377	.427	.464			.194	.262	.302	.359	.402	.448	.494
400,000		.170	[b] .222	.276	.308	.353	.390	.426	.468		[a] .165	.227	.291	[c] .331	.380	.423	.476	.515			.232	.300	[c] .342	.400	.445	.494	.542
300,000	[a] .197	.232	.285	[c] .340	.373	.420	.460	.497	.540	[a] .188	.229	[b] .291	[c] .355	.396	.447	.494	.547	.588		[a] .226	[b] .295	[c] .363	.406	.466	.516	.566	.615
250,000	.250	[b] .285	.339	.394	.428	.477	.519	.560	.604	.238	.280	.342	.407	.448	.503	.551	.605	.647	[a] .233	.279	.348	.416	.459	.522	.575	.627	.677
0000	[b] .302	.337	[c] .391	.447	.482	.534	.579	.623	.669	.289	[b] .331	.395	.462	.505	.562	.611	.668	.712	.285	[b] .331	.401	.470	.516	.580	.634	.687	.740
000	.399	.436	.494	.552	.588	.644	.689	.733	.779	[b] .385	.421	[c] .491	.560	.607	.668	.719	.778	.823	[b] .381	.427	[c] .498	.568	.618	.684	.740	.797	.851
00	.518	[c] .555	.613	.672	.709	.767	.814	.850	.909	.505	.548	.612	.682	.730	.795	.848	.908	.953	.502	.548	.620	.690	.743	.817	.873	.932	.988
0	[c] .668	.705	.766	.827	.869	.928	.975	1.02	1.07	.655	[c] .696	.764	.836	.885	.954	1.01	1.07	1.11	.648	[c] .694	.771	.845	.903	.980	1.03	1.09	1.15
1	.853	.890	.957	1.03	1.07	1.13	1.18	1.23	1.28	[a] .844	.885	.957	1.03	1.08	1.15	1.21	1.28	1.32	[c] .837	.885	.954	1.04	1.10	1.18	1.23	1.29	1.36
2	1.10	1.13	1.20	1.28	1.32	1.38	1.43	1.48	1.54	1.09	1.13	1.20	1.28	1.33	1.40	1.46	1.53	1.58	1.08	1.13	1.20	1.29	1.35	1.43	1.49	1.55	1.62
3	1.41	1.44	1.51	1.59	1.63	1.69	1.75	1.80	1.87	1.40	1.44	1.51	1.59	1.65	1.72	1.78	1.85	1.90	1.39	1.44	1.51	1.60	1.66	1.74	1.81	1.87	1.94
4	1.80	1.83	1.90	1.98	2.02	2.09	2.15	2.21	2.28	1.78	1.82	1.90	1.98	2.04	2.11	2.18	2.25	2.31	1.77	1.82	1.90	1.99	2.06	2.14	2.21	2.27	2.35
	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]	[d]

Table 26. — Values of *M* for Overhead Aluminum Wires at 60 Cycles per Second.

Cir. Mils, or A.W.G.	Wires on 18-Inch Centers.									Wires on 36-Inch Centers.									Wires on 60-Inch Centers.								
	Power-Factor of Load in Per Cent.																										
	Lead.			Lag.						Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
1,000,000	....	....	....	.200[a]	.245[a]	.307[b]	.352[b]	.403[c]	.451[c]	....	....	....	.223[a]	.276[a]	.346[b]	.402[b]	.463[c]	.517[c]	....	....	....	.239[a]	.297[a]	.377[b]	.438[b]	.506[c]	.571[c]
800,000	....	....	....	.220	.266[b]	.329	.376[c]	.428	.478	....	....	....	.243	.297	.368	.425	.488	.544	....	....	....	.261	.319	.399	.462	.531	.598
700,000	....	....	....	.234	.282	.346	.394	.446	.498	....	....	....	.257	.311[b]	.384	.442	.506	.564	....	....	....	.275	.335	.416	.480	.550	.612
600,000	....	....	.177[a]	.257	.306	.374	.419	.472	.525	....	....	....	.279	.334	.410	.468[c]	.533	.591	....	....	....	.296	.357[b]	.439	.503[c]	.574	.644
500,000	....	....	.201	.282[b]	.332	.396[c]	.445	.500	.557	....	....	.210[a]	.306	.362	.439	.497	.563	.623	....	....	....	.322	.384	.466	.530	.602	.675
400,000	....	....	.240	.324	.375	.441	.492	.550	.608	....	....	.249	.347[b]	.406	.484[c]	.543	.601	.675	....	....	.256[a]	.360[b]	.424	.508	.574	.647	.722
300,000	....	.220[a]	.303	.388	.440[c]	.506	.559	.618	.679	....	....	.312	.410	.470	.549	.610	.669	.747	....	....	.318	.424	.488	.573[c]	.640	.714	.790
250,000	....	.270	.353[b]	.438[c]	.491	.560	.616	.677	.738	....	.265[a]	.364	.462	.522[c]	.602	.668	.728	.807	....	.261[a]	.368	.477	.542[c]	.629	.700	.774	.850
0000	.270[a]	.324	.407	.492	.546	.619	.679	.745	.807	....	.320	.419[b]	.518	.579	.661	.729	.791	.875	....	.316	.425[b]	.534	.600	.689	.761	.836	.914
000	.366	.420[b]	.505	.590	.647	.729	.792	.862	.925	.347[a]	.415	.515	.616[c]	.679	.767	.842	.905	.991	.342[a]	.410	.520	.630[c]	.701	.795	.876	.951	1.03
00	.484[b]	.538	.623[c]	.710	.770	.856	.925	.995	1.06	.465	.538[b]	.634	.736	.802	.895	.975	1.04	1.13	.462	.530[b]	.640	.751	.826	.925	1.01	1.09	1.17
0	.630	.684	.773	.868	.930	1.02	1.09	1.16	1.23	.609[b]	.678	.781[c]	.887	.956	1.05	1.13	1.20	1.29	.607[b]	.677	.793[c]	.904	.981	1.08	1.17	1.26	1.34
1	.872	.876[c]	.970	1.07	1.13	1.22	1.29	1.37	1.44	.802	.873[c]	.978	1.09	1.16	1.26	1.34	1.41	1.50	.794	.864	.983	1.10	1.18	1.28	1.37	1.47	1.55
2	1.06[c]	1.11	1.21	1.32	1.38	1.47	1.54	1.62	1.70	1.04	1.11	1.22	1.33	1.41	1.51	1.60	1.67	1.76	1.03	1.10[c]	1.22	1.34	1.42	1.53	1.62	1.72	1.81
3	1.37	1.42	1.52	1.63	1.69	1.79	1.86	1.94	2.03	1.35[c]	1.42	1.53	1.64	1.72	1.82	1.91	1.99	2.09	1.34[c]	1.41	1.53	1.66	1.74	1.85	1.94	2.04	2.13
4	1.75	1.80	1.91	2.02	2.09	2.20	2.27	2.35	2.44	1.73	1.80	1.92	2.03	2.11	2.22	2.31	2.40	2.50	1.72	1.79	1.92	2.05	2.13	2.25	2.34	2.45	2.54
	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d	d

CHAPTER IV.

ALTERNATING-CURRENT TRANSMISSION BY UNDERGROUND CABLES.

44. Introduction. — The calculations for underground cables are very similar to those for overhead lines. The main difference in results is due to the proximity of the wires, in consequence of which inductance effects are considerably reduced. However, with commercial thickness of insulation such effects are not entirely eliminated for the larger wires at the higher frequencies. The difference may be ascertained by comparing the values of M at 100 per cent power-factor in Tables 34 and 35 with the resistance per mile of the same wires in Table 3. As the conductors in underground cables are almost always copper, the results have been calculated for that metal alone. The calculations for 15 cycles per second have been omitted.

The error of the common assumption that the resistance drop equals the volt loss is indicated in Table 28 below for insulation of $\frac{10}{64}$ inch thickness.

Table 28. — Error in Per Cent of True Volt Loss, Assuming Resistance Drop = Volt Loss.

Size of Wire. A.W.G.	25 Cycles per Sec.			60 Cycles per Sec.			125 Cycles per Sec.		
	Lagging Power Factor of Load in Per Cent.								
	100	90	80	100	90	80	100	90	80
0000	0	−1	+ 2	−2	−15	−15	−8	−35	−39
0	0	+3	+10	0	− 4	− 1	−3	−18	−19
4	0	+7	+15	0	+ 2	+ 8	0	− 5	− 2

It is obvious from the table that the values resulting from the assumption may be much too high or too low, except near 100 per cent power-factor. The error is due to the power-factor angles of the load and line. (See Figs. 1 and 2, page 30.)

45. Maximum Error. — The formulas apply for all volt losses

less than 20 per cent of the generated volts or 25 per cent of the voltage at load. As in the case of overhead lines the formulas are approximate, but the error in general is much reduced, even without subdivision of the wire factors into sections. The maximum error in the calculation for the per cent volt loss is about 5 per cent near 0 per cent and 20 per cent volt losses, but it gradually reduces to a zero error near 10 per cent loss. Thus for a true volt loss of 5 per cent the extreme range of the calculated result is about 4.85 to 5.15 per cent. Under ordinary conditions the error is immeasurably small.

The maximum errors of the formulas under the most severe conditions are shown in the following table.

Table 29. — Maximum Error in Per Cent of True Values, at 20° Cent.

Items.	For Voltage Given at Load.	For Voltage Given at Source.
Volt loss or per cent volt loss	5	5
Kilovolts at load or source	1	1
Power loss or per cent power loss	0	2
Kilowatts at source	0	0
Amperes per wire	0	1
Power-factor at source	1	1

46. Temperature. — The values of M have been calculated at 20° cent. or 68° fahr. The effect of temperature variations is introduced in the calculations by means of A, Table 32, page 72.

47. Properties of Conductors. — The values of A are given for copper of 100 per cent Matthiessen's standard, but any other conductivity may be used by dividing A of Table 32 by the given value in the standard scale. It has been assumed that conductors smaller than No. 3 are solid and those larger are stranded. However, any practical variation from this assumption will have no appreciable effect on any of the calculations.

48. Thickness of Insulation. — The values of M have been calculated for a thickness of insulation of $\frac{10}{64}$ inch on each conductor, but any commercial variation will not materially affect the results.

49. Current-carrying Capacity. — The current-carrying capacity in amperes per wire is shown in Table 4, page 8, for single

and three-conductor cables. In most cases the size of cable is determined only by the volt loss, but in some problems the current-carrying capacity may be the ruling factor. Hence it is advisable to note from Table 4 that the calculated wire is capable of carrying the required current without undue heating.

50. Capacity Effects. — Although underground lines are of comparatively short length, yet the closeness of the conductors and the high values of the specific inductive capacity of the insulation sometimes make it advisable to determine the capacity values. The method of including capacity in the calculation of volt loss, power loss and the power-factor at source is exactly like that given for overhead lines in paragraph 42, page 38. The capacity factor, c, for underground cables is to be taken from Table 30 below. The specific inductive capacity was assumed to be 2.5, but for any other value, c in Table 30 should be multiplied by the ratio of the given value to 2.5. Thus c for a three-phase cable having No. 00 conductors and a specific inductive capacity of 3, is

$$\frac{0.0007 \times 3}{2.5} = 0.00084.$$

Table 30. — Values of c for Underground Cables.

Circular Mils, or A.W.G.	1-Phase Cable.	3-Phase Cable.
300,000 to 0	0.0003	0.0007
1 to 6	0.0003	0.0006

For specific inductive capacity of insulation other than 2.5, multiply c by given value and divide by 2.5.

Owing to upper harmonics the charging current in cables may be as much as 40 per cent greater than calculated for a sine-wave e. m. f. However, the effect cannot be included in a general way as the increase depends entirely upon the generator design.

51. Examples. — The items to be specified are the same as stated in paragraph 38, page 37, for overhead lines. The formulas and the method of using them are also similar except where the per cent volt loss is involved. In such cases V is used directly when the voltage at load is given, while V_0''' from Table 31, page 72, is used in formulas (66) and (70) where the voltage at

source is specified. The meaning of the term "source" is specified in paragraph 32, page 35. Following are typical examples illustrating the application of the formulas on page 71.

Example 21. — A one-phase underground cable 5 miles long is to deliver 800 kw. at 98 per cent lagging power-factor and 60 cycles per second with a loss of 10 per cent of the 6600 generated volts, with copper of 100 per cent conductivity and a temperature of 40° cent.

GIVEN ITEMS.

$W = 800$ kw.; $E_0 = 6.6$ kv.; $V_0 = 10$ per cent;
$L = 5$ miles; $K = 0.98$ lag.

From Table 32, page 72, $A = 0.215$.
From Table 31, page 72, $V_0''' = 9.0$.

REQUIRED ITEMS.

Size of each wire. — From (66),

$$M = \frac{(6.6)^2 \times 9}{0.215 \times 800 \times 5} = 0.456.$$

From Table 35, page 74, use No. 00, for which $M = 0.448$.
Per cent volt loss. — From (70),

$$V_0''' = \frac{0.448 \times 0.215 \times .8005}{(6.6)^2} = 8.84.$$

From Table 31, page 72, $V_0 = 9.84$ per cent.
Kilovolts at load. — From (72), $E = 6.6\ (1 - 0.0984) = 5.95$ kv.
Volt loss. — From (73), $v = 1000\ (6.60 - 5.95) = 650$ volts.
Per cent power loss. — From Table 3, page 7, $R = 0.411$ ohm.
From (75),

$$P = \frac{0.411 \times 0.215 \times 800 \times 5}{[6.6\ (1 - 0.0984)\ 0.98]^2} = 10.4 \text{ per cent.}$$

Kilowatts at source. — From (76),

$$W_0 = 800\ (1 + 0.104) = 883 \text{ kw.}$$

Watt loss. — From (77), $p = 1000\ (883 - 800) = 83{,}000$ watts.
Amperes per wire. — From Table 33, page 72, $B = 1.000$.
From (79),

$$I = \frac{800}{6.6\ (1 - 0.0984)\ 0.98} = 137 \text{ amperes.}$$

Power-factor at source. — From (80),

$$K_0 = (1 + 0.104)\ (1 - 0.0984)\ 0.98 = 0.975.$$

Example 22. — A one-phase underground cable with No. 1 copper conductors of 98 per cent conductivity and 10,000 feet long, delivers 150 kw. at 85 per cent lagging power-factor and 125 cycles per second. The generator voltage is 2200 and the temperature is assumed 50° cent.

GIVEN ITEMS.

$W = 150$ kw.; $E_0 = 2.2$ kv.; $L = 10{,}000$ feet; $K = 0.85$ lag.

From Table 35, page 74, $M = 0.896$.

From Table 32, page 72, $A = \frac{0.0423}{0.98} = 0.0432$.

REQUIRED ITEMS.

Per cent volt loss. — From (70),

$$V_0''' = \frac{0.896 \times 0.0432 \times 150 \times 10}{(2.2)^2} = 12.0.$$

From Table 31, page 72, $V_0 = 14.0$ per cent.

Kilovolts at load. — From (72), $E = 2.2\ (1 - 0.14) = 1.89$ kv.

Example 23. — A three-phase load of 3000 kw. and 25 cycles per second is to be delivered to a receiver having a lagging power-factor of 95 per cent, over 6 miles of underground cable of 97 per cent conductivity with a loss of 5 per cent of the 11,000 volts generated; temperature 20° cent.

GIVEN ITEMS.

$W = 3000$ kw.; $E_0 = 11$ kv.; $V_0 = 5$ per cent;
$L = 6$ miles; $K = 0.95$ lag.

From Table 31, page 72, $V_0''' = 4.8$.

From Table 32, page 72, $A = \frac{0.100}{0.97} = 0.103$.

REQUIRED ITEMS.

Size of each wire. — From (66),

$$M = \frac{(11)^2 \times 4.8}{0.103 \times 3000 \times 6} = 0.313.$$

From Table 34, page 73, use No. 0000, for which $M = 0.280$.
Per cent volt loss. — From (70),

$$V_0''' = \frac{0.280 \times 0.103 \times 3000 \times 6}{(11)^2} = 4.30.$$

From Table 31, page 72, $V_0 = 4.50$ per cent.
Volt loss. — From (73), $v = 10 \times 4.5 \times 11 = 495$ volts.
Per cent power loss. — From Table 3, page 7, $R = 0.258$ ohm.
From (75),

$$P = \frac{0.258 \times 0.103 \times 3000 \times 6}{[11\ (1 - 0.045)\ 0.95]^2} = 4.80 \text{ per cent.}$$

Amperes per wire. — From Table 33, page 72, $B = 0.578$.
From (79),

$$I = \frac{0.578 \times 3000}{11\ (1 - 0.045)\ 0.95} = 174 \text{ amperes.}$$

From Table 4, page 8, it is seen that the cable will not overheat.

Power-factor at source. — From (80),

$$K_0 = (1 + 0.048)\ (1 - 0.045)\ 0.95 = 0.95.$$

Example 24. — A three-phase cable with No. 0 conductors of 98 per cent conductivity delivers 2500 kw. at 6000 volts, 95 per cent leading power-factor and 60 cycles per second, to a receiver 3 miles from the generator, the temperature of the wires being 40° cent.

GIVEN ITEMS.

$W = 1500$ kw.; $E = 6$ kv.; $L = 5$ miles; $K = 0.95$ lead.
From Table 35, page 74, $M = 0.475$.
From Table 32, page 72, $A = \frac{0.108}{0.98} = 0.110$.

REQUIRED ITEMS.

Per cent volt loss. — From (69),

$$V = \frac{0.475 \times 0.11 \times 1500 \times 5}{(6)^2} = 10.9 \text{ per cent.}$$

Kilovolts at source. — From (71),

$$E_0 = 6\ (1 + 0.109) = 6.65 \text{ kv.}$$

Per cent power loss. — From Table 3, page 7, $R = 0.518$ ohm.

From (74),

$$P = \frac{0.518 \times 0.11 \times 1500 \times 5}{(6)^2\ (0.95)^2} = 13.2 \text{ per cent.}$$

Amperes per wire. — From Table 33, page 72, $B = 0.578$.

From (78),

$$I = \frac{0.578 \times 1500}{6 \times 0.95} = 140 \text{ amperes.}$$

From Table 4, page 8, this current is slightly in excess of the safe current-carrying capacity of the cable.

Power-factor at source. — From (80),

$$K_0 = \frac{(1 + 0.132)\,0.95}{1 + 0.109} = 0.970.$$

There is no indication from the above result whether the current at the source is leading or lagging. It then becomes necessary to construct a vector diagram similar to Fig. 2, page 30, from which it will be found that the current in the above case is leading at the source.

Example 25. — A three-phase load of 400 kw. at 95 per cent lagging power-factor and 125 cycles per second is to be transmitted 8 miles with a loss of 7.5 per cent of the generated voltage, 6600. Include capacity effects and assume wires of 97 per cent conductivity and a temperature of 30° cent.

GIVEN ITEMS.

$W = 400$ kw.; $E_0 = 6.6$ kv.; $V_0 = 7.5$ per cent;
$L = 8$ miles; $K = 0.95$ lag.

From Table 31, page 72, $V_0''' = 6.95$.

From Table 32, page 72, $A = \frac{0.104}{0.97} = 0.107$

REQUIRED ITEMS.

Apparent power-factor of load. — Assuming that the required size of conductors will be between Nos. 1 and 6, $c = 0.0006$ (from Table 30, page 65).

From Table 14, page 39, the reactance factor for 95 per cent lagging power-factor = 0.33.

From (44),

$$t' = 0.33 - \frac{0.0006\,[6.6\,(1 - 0.075)]^2\,125 \times 8}{400} = 0.27.$$

By interpolation in Table 14, page 39, the apparent power-factor of load, K', = 0.97 lag.

Size of each wire. — From (66),

$$M = \frac{(6.6)^2 \times 6.95}{0.107 \times 400 \times 8} = 0.885.$$

From Table 35, page 74, use No. 2 cable, for which

$$M = 0.925, \text{ approximately.}$$

Per cent volt loss. — From (70),

$$V_0''' = \frac{0.925 \times 0.107 \times 400 \times 8}{(6.6)^2} = 7.27.$$

From Table 31, page 72, $V_0 = 7.87$ per cent.

Per cent power loss. — From Table 3, page 7, $R = 0.824$ ohm. From (75),

$$P = \frac{0.824 \times 0.107 \times 400 \times 8}{[6.6\,(1 - 0.0787)\,0.97]^2} = 8.10 \text{ per cent.}$$

Kilowatts at source. —From (76), $W_0 = 400\,(1 + 0.081) = 432$ kw.

Power-factor at source (not including capacity). — From (80),

$$K_0' = (1 + 0.081)\,(1 - 0.0727)\,0.97 = 0.97 \text{ lag.}$$

From Table 14, page 39, $t_0' = 0.27$.

From (45), page 38,

$$t_0 = 0.27 - \frac{0.0006\,(6.6)^2\,125 \times 8}{432} = 0.21.$$

From Table 14, page 39, the true power-factor at load, K_0 (corresponding to t_0), = 0.98 lag.

Formulas for A. C. Transmission by Underground Cables.

Required Items.	For Voltage Given at Load.	For Voltage Given at Source.
Size of each wire from per cent volt loss.	*A* from Table 32, page 72.	
	$M = \frac{E^2V}{AWL}$ . . . . (65)	V_0''' from Table 31, page 72. $M = \frac{E_0^2V_0'''}{AWL}$ . . . (66)
	Then find wire from Table 34 or 35, page 73 or 74.	
Size of each wire from per cent power loss.	$R = \frac{PE^2K^2}{AWL}$ . . . (67)	$R = \frac{P[E_0(1-0.01V_0)K]^2}{AWL}$ (68)
	Then find wire from Table 3, page 7.	
per cent volt loss.	$V = \frac{MAWL}{E^2}$ . . (69)	$V_0''' = \frac{MAWL}{E_0^2}$ . . (70)
		V_0 from Table 31, page 72.
Kilovolts at source or load.	$E_0 = E(1+0.01V)$. (71)	$E = E_0(1-0.01V_0)$. (72)
Volt loss.	$v = 1000(E_0 - E) = 10\,VE = 10\,V_0E_0$ . . . (73)	
per cent power loss.	*R* from Table 3, page 7.	
	$P = \frac{RAWL}{E^2K^2}$ . . . (74)	$P = \frac{RAWL}{[E_0(1-0.01V_0)K]^2}$ (75)
Kilowatts at source.	$W_0 = W(1+0.01P)$ . . . . . (76)	
Watt loss.	$p = 1000(W_0 - W) = 10\,PW$ . . . (77)	
Amperes per wire. (See par. 34, page 35.)	*B* from Table 33, page 72.	
	$I = \frac{BW}{EK}$ . . . . (78)	$I = \frac{BW}{E_0(1-0.01V_0)K}$. (79)
Power-factor at source.	$K_0 = \frac{(1+0.01P)K}{1+0.01V} = (1+0.01P)(1-0.01V_0)K$. (80)	

When given E and V_0, find V from $V = V_0/(1-0.01V_0)$ . . . . . (81)
When given E_0 and V, find V_0 from $V_0 = V/(1+0.01V)$ . . . . . (82)
When given W', find W from $W = W'K$ . . . . . (83)

NOTATION.

A = Transmission factor. Table 32, page 72.
B = Transmission constant. Table 33, p. 72.
E = Kilovolts (1000 volts) between wires at load. See par. 32, p. 35.
E_0 = Kilovolts (1000 volts) between wires at source. See par. 32, p. 35.
I = Amperes per wire. See par. 34, p. 35.
K = Power-factor of load, in decimals.
K_0 = Power-factor at source, in decimals.
L = Distance from source to load, in miles or 1000 ft.
M = Wire factor, Tables 34 and 35, pp. 73 and 74.
P = Power loss in per cent of power at load.
p = Power loss in watts.
R = Resistance per mile of one wire, in ohms. Table 3, p. 7.
V = Volt loss in per cent of volts at load.
V_0 = Volt loss in per cent of volts at source.
V_0''' = Volt loss factor. Table 31, p. 72.
v = Volt loss (volts at source — volts a load).
W = Real kilowatts at load.
W_0 = Real kilowatts at source.
W' = Kilovolt-amperes (apparent power) at load.

Table 31. — Values of $V_0''' = V_0\ (1 - 0.01\ V_0)$.

V_0	V_0'''	V_0	V_0'''	V_0	V_0'''	V_0	V_0'''	V_0	V_0'''	V_0	V_0'''	V_0	V_0'''
0.0	0.0	2.0	2.0	4.0	3.8	6.0	5.6	8.0	7.4	10.0	9.0	15.0	12.8
0.2	0.2	2.2	2.1	4.2	4.0	6.2	5.8	8.2	7.5	10.5	9.4	15.5	13.1
0.4	0.4	2.4	2.3	4.4	4.2	6.4	6.0	8.4	7.7	11.0	9.8	16.0	13.4
0.6	0.6	2.6	2.5	4.6	4.4	6.6	6.2	8.6	7.9	11.5	10.2	16.5	13.8
0.8	0.8	2.8	2.7	4.8	4.6	6.8	6.3	8.8	8.0	12.0	10.6	17.0	14.1
1.0	1.0	3.0	2.9	5.0	4.8	7.0	6.5	9.0	8.2	12.5	10.9	17.5	14.4
1.2	1.2	3.2	3.1	5.2	4.9	7.2	6.7	9.2	8.4	13.0	11.3	18.0	14.8
1.4	1.4	3.4	3.3	5.4	5.1	7.4	6.9	9.4	8.5	13.5	11.7	18.5	15.1
1.6	1.6	3.6	3.5	5.6	5.3	7.6	7.0	9.6	8.7	14.0	12.0	19.0	15.4
1.8	1.8	3.8	3.7	5.8	5.5	7.8	7.2	9.8	8.8	14.5	12.4	20.0	16.0

Table 32. — Values of *A* for Balanced Loads.

Temperature in Degrees.		1-Phase System with 2 Equal Wires.		2-Phase System with 3 Equal Wires.		2-Phase System with 4 Equal Wires, or 3-Phase System with 3 Equal Wires.	
Cent.	Fahr.	For *L* in Miles.	For *L* in 1000 ft.	For *L* in Miles.	For *L* in 1000 ft.	For *L* in Miles.	For *L* in 1000 ft.
0	32	0.185	0.0351	0.112	0.0212	0.0926	0.0175
10	50	0.193	0.0365	0.116	0.0220	0.0963	0.0182
20	68	0.200	0.0379	0.121	0.0229	0.100	0.0190
30	86	0.208	0.0393	0.125	0.0237	0.104	0.0197
40	104	0.215	0.0408	0.130	0.0246	0.108	0.0204
50	122	0.223	0.0423	0.135	0.0255	0.112	0.0212

The above values are for copper wires of 100 per cent conductivity and aluminum wires of 62 per cent conductivity, in Matthiessen's standard scale.

For copper of other conductivity, divide *A* by given conductivity.

For aluminum by other conductivity, divide *A* by given conductivity and multiply by 0.62.

Table 33. — Values of *B* for Balanced Loads.

1-Phase System.	2-Phase System.	3-Phase System.
1.000	0.500	0.578

Table 34. — Values of *M* for Multiple Conductor Copper Cables.

Cir. Mils, or A.W.G.	25 Cycles per Second.									40 Cycles per Second.								
	Power-Factor of Load in Per Cent.																	
	Lead.			Lag.						Lead.			Lag.					
	95	98	100	98	95	90	85	80	75	95	98	100	98	95	90	85	80	75
300,000	0.166	0.172	0.182	0.195	0.204	0.214	0.223	0.234	0.243	0.158	0.168	0.186	0.205	0.217	0.233	0.248	0.262	0.276
250,000	0.203	0.209	0.219	0.232	0.241	0.251	0.261	0.272	0.281	0.194	0.204	0.222	0.242	0.254	0.271	0.296	0.301	0.316
0000	0.242	0.248	0.258	0.271	0.280	0.291	0.302	0.314	0.324	0.232	0.242	0.261	0.281	0.295	0.312	0.327	0.343	0.358
000	0.309	0.315	0.326	0.340	0.349	0.360	0.372	0.385	0.395	0.299	0.309	0.328	0.349	0.363	0.381	0.397	0.415	0.431
00	0.393	0.399	0.411	0.426	0.435	0.447	0.459	0.474	0.485	0.383	0.393	0.413	0.435	0.450	0.468	0.487	0.506	0.522
0	0.500	0.506	0.518	0.534	0.544	0.557	0.569	0.586	0.600	0.489	0.499	0.520	0.543	0.558	0.577	0.596	0.618	0.637
1	0.635	0.641	0.653	0.670	0.681	0.695	0.709	0.727	0.742	0.624	0.634	0.655	0.679	0.695	0.716	0.735	0.761	0.780
2	0.805	0.812	0.824	0.841	0.852	0.868	0.883	0.904	0.924	0.793	0.803	0.824	0.850	0.866	0.888	0.909	0.939	0.959
3	1.02	1.03	1.04	1.06	1.07	1.09	1.10	1.13	1.15	1.01	1.02	1.04	1.07	1.09	1.11	1.13	1.16	1.19
4	1.29	1.30	1.31	1.33	1.34	1.36	1.38	1.42	1.44	1.27	1.28	1.31	1.34	1.36	1.38	1.42	1.46	1.49
5	1.63	1.64	1.65	1.68	1.69	1.71	1.73	1.78	1.80	1.61	1.62	1.65	1.68	1.70	1.73	1.77	1.82	1.85
6	2.06	2.07	2.08	2.11	2.12	2.15	2.17	2.22	2.25	2.04	2.05	2.08	2.12	2.14	2.17	2.21	2.26	2.30
8	3.29	3.31	3.31	3.35	3.36	3.40	3.43	3.50	3.54	3.27	3.29	3.31	3.36	3.38	3.42	3.47	3.54	3.60
10	5.24	5.26	5.27	5.31	5.32	5.36	5.41	5.52	5.58	5.22	5.24	5.27	5.32	5.34	5.39	5.45	5.56	5.64

Table 35. — Values of *M* for Multiple Conductor Copper Cables.

Cir. Mils, or A.W.G.	60 Cycles per Second.									125 Cycles per Second.						
	Power-Factor of Load in Per Cent.															
	Lead.			Lag.						Lag.						
	95	98	100	98	95	90	85	80	75	100	98	95	90	85	80	75
300,000	.....	.....	0.190	0.220	0.238	0.262	0.283	0.303	0.326	.....	.....	0.311	0.369	0.413	0.457	0.501
250,000	.....	0.198	0.226	0.256	0.275	0.300	0.321	0.342	0.365	.....	0.308	0.347	0.405	0.450	0.494	0.539
0000	.....	0.237	0.265	0.296	0.315	0.340	0.362	0.384	0.407	.....	0.346	0.385	0.443	0.488	0.533	0.579
000	0.287	0.302	0.330	0.362	0.382	0.409	0.433	0.456	0.480	0.346	0.409	0.449	0.508	0.554	0.600	0.648
00	0.370	0.386	0.415	0.448	0.468	0.496	0.520	0.545	0.570	0.430	0.496	0.537	0.597	0.643	0.690	0.741
0	0.475	0.491	0.520	0.550	0.575	0.605	0.630	0.660	0.686	0.533	0.601	0.642	0.704	0.752	0.799	0.851
1	0.699	0.625	0.655	0.691	0.714	0.745	0.772	0.803	0.830	0.665	0.738	0.782	0.845	0.896	0.945	1.00
2	0.777	0.792	0.825	0.863	0.887	0.920	0.948	0.981	1.01	0.834	0.910	0.956	1.02	1.08	1.13	1.19
3	1.00	1.02	1.04	1.08	1.10	1.14	1.17	1.21	1.24	1.05	1.13	1.18	1.25	1.31	1.37	1.43
4	1.26	1.28	1.31	1.35	1.38	1.42	1.46	1.51	1.54	1.31	1.40	1.46	1.53	1.60	1.67	1.73
5	1.60	1.62	1.65	1.70	1.73	1.77	1.81	1.86	1.90	1.65	1.75	1.81	1.89	1.96	2.04	2.11
6	2.02	2.04	2.08	2.13	2.16	2.21	2.25	2.32	2.36	2.08	2.19	2.25	2.33	2.41	2.50	2.57
8	3.25	3.28	3.31	3.37	3.40	3.46	3.51	3.60	3.66	3.31	3.43	3.50	3.59	3.67	3.80	3.89
10	5.20	5.23	5.27	5.34	5.38	5.45	5.51	5.63	5.72	5.27	5.39	5.46	5.57	5.69	5.84	5.95

CHAPTER V.

INTERIOR WIRES FOR ALTERNATING-CURRENT DISTRIBUTION.

52. **Introduction.** — Interior wiring involves short runs of conductors; therefore the conductors are often determined by their current-carrying capacity rather than by conditions of maximum drop. It is generally advisable to note the required size of wire for both conditions. Table 4, page 8, gives the National Electric Code Standard for current-carrying capacity of interior wires.

The formulas for interior wiring calculations are similar to the preceding ones for alternating-current transmission. The units of power, voltage and distance have been changed in order to facilitate calculation. All the required items are expressed in terms of the current per wire and the per cent volt loss is also given in terms of the power at load so that problems involving watts at load and voltage at source may be solved without preliminary approximation.

The ordinary error of the calculation is considerably less than stated in paragraph 45 for underground conductors, on account of the smaller wires usually employed. The error at 20° cent. of the common assumption that the resistance drop equals the volt loss is indicated in Table 36, page 76, for wires in conduits and on 3-inch centers.

It is apparent that results based on the assumptions of Table 36 may be much greater or much less than the true values, even at power-factors near unity. The error is due to the power-factor angles of the load and line. (See Figs. 1 and 2, page 30.)

53. **Properties of Conductors.** — The values of M have been calculated for wires of 100 per cent Matthiessen's Standard at a temperature of 20° cent. or 68° fahr. However, the effects of temperature and conductivity are introduced by means of a and b in Table 37, page 82.

54. **Spacing of Wires.** — Table 39, page 83, gives the values of M for wires with a thickness of insulation of $\frac{10}{64}$ inch, while

Table 40, page 84, is calculated for conductors on 3-inch centers. Any slight variation from either of these spacings will not appreciably alter the results, even for the largest wires at the highest commercial frequency.

Table 39 is to be used for wires in conduit, duplex cables, multi-conductor cables or twisted wires. Table 40 is for wires in molding, or for open work such as wires on cleats or knobs.

55. Ampere-Feet. — A simple method of dealing with distributed lamps is by use of formula (84) on page 81. The term Il is the ampere-feet and is equal to the sum of the products given by each load, I multiplied by its distance l.

Table 36. — Error in Per Cent of True Volt Loss, Assuming Resistance Drop = Volt Loss.

Size of Wire. A.W.G.	60 Cycles per Sec.			125 Cycles per Sec.		
	Lagging Power-Factor of Load in Per Cent.					
	100	90	80	100	90	80
Wires in Conduit.						
0000	−2	−15	−15	−8	−35	−39
4	0	+ 2	+ 8	0	− 5	− 2
14	0	+ 8	+18	0	+ 7	+16
Wires on 3-Inch Centers.						
0000	−7	−34	−37	...	−54	−59
4	−0	−4	− 1	−2	−16	−16
14	0	+8	+17	0	+ 6	+14

56. Examples. — The application of formulas on page 81 are illustrated in the following examples. It should be carefully noted that the per cent volt and power losses are expressed as whole numbers and that l is the distance of transmission from source to load, which is the same as the length of one wire. In formulas (90) and (91), r is the resistance per foot of one wire, equal to the values in Table 3, column 4, divided by 1000. The meaning of the term source is stated in paragraph 32, page 35.

Example 26. — A group of lamps having a power-factor of 95 per cent lag is to be supplied with 20 amperes at 60 cycles per second over a single-phase copper cable 150 feet long. Calculate the size of wire for 2 volts drop at 30° cent. for copper of 97 per cent conductivity.

GIVEN ITEMS.

$I = 20$ amperes; $v = 2$ volts; $l = 150$ feet; $K = 0.95$ lag.
From Table 37, page 82, $a = 963 \times 0.97 = 934$.

REQUIRED ITEMS.

Size of each wire. — From (84),

$$M = \frac{934 \times 2}{20 \times 150} = 0.623.$$

From Table 39, page 83, use No. 5, for which $M = 0.623$.

Example 27. — At 40° cent. a single-phase cable with No. 8 conductors of 97 per cent conductivity and 200 feet long, delivers 25 amperes at 98 per cent lagging power-factor from a transformer which gives 100 volts at 125 cycles per second.

GIVEN ITEMS.

$I = 25$ amperes; $e_0 = 100$ volts; $l = 200$ feet; $K = 0.98$ lag.
From Table 39, page 83, $M = 1.27$.
From Table 37, page 82,

$$a = 928 \times 0.97 = 900; \quad b = \frac{215}{0.97} = 222.$$

REQUIRED ITEMS.

Volt loss. — From (85),

$$v = \frac{1.27 \times 25 \times 200}{900} = 7.0 \text{ volts.}$$

Volts at load. — From (89), $e = 100 - 7 = 93$ volts.
Per cent volt loss. — From (86),

$$V = \frac{7}{0.93} = 7.5 \text{ per cent.}$$

Per cent power loss. — From Table 3, page 7, $r = 0.000627$.

From (90),

$$P = \frac{0.000627 \times 222 \times 25 \times 200}{93 \times 0.98} = 7.6 \text{ per cent.}$$

Watts at load. — From Table 38, page 82, $B = 1.000$.

From (94), $$w = \frac{25 \times 93 \times 0.98}{1.00} = 2280 \text{ watts.}$$

Watt loss. — From (92), $p = 0.076 \times 2280 = 173$ watts.
Power-factor at source. — From (96),

$$K_0 = \frac{(1 + 0.076)\, 0.98}{1 + 0.075} = 0.98 \text{ lag.}$$

Example 28. — A 60-cycle step-down transformer with 120 volts at its secondary terminals is connected to a load of 2000 watts at 90 per cent lagging power-factor over 200 feet of single-phase circuit on cleats. Determine the size of wire for 5 per cent drop at 20° cent. and copper of 98 per cent conductivity.

GIVEN ITEMS.

$w = 2000$ watts; $e_0 = 120$ volts; $V_0 = 5$ per cent;
$l = 200$ feet; $K = 0.90$ lag.

From Table 31, page 72, $V_0''' = 4.8$.
From (98), $v = 0.05 \times 120 = 6$ volts.
From Table 37, page 82, $a = 1000 \times 0.98 = 980$.
From (95), (in which B from Table 38, page 82, $= 1.000$),

$$I = \frac{2000}{120\,(1 - 0.05)\,0.90} = 19.5 \text{ amperes.}$$

REQUIRED ITEMS.

Size of each wire. — From (84),

$$M = \frac{980 \times 6}{19.5 \times 200} = 1.51.$$

From Table 40, page 84, use No. 8, for which $M = 1.20$.
Per cent volt loss. — From (87),

$$V_0''' = \frac{1.20 \times 2000 \times 200}{0.01 \times 980 \times (120)^2} = 3.40.$$

From Table 31, page 72, $V_0 = 3.50$ per cent.

Volts at load. — From (89), $e = 120\ (1 - 0.035) = 116$ volts.

Amperes per wire. — From (94),

$$I = \frac{2000}{116 \times 0.90} = 19.2 \text{ amperes.}$$

From Table 4, page 8, it is seen that the current will be safely carried by No. 8 wires.

Example 29. — A load of 1500 watts at 110 volts, 85 per cent lagging power-factor and 60 cycles per second is to be delivered 250 feet over a two-phase four-wire circuit on cleats, with a loss of 2 per cent of the generated volts. Calculate the size of wire of 98 per cent conductivity for a temperature of 30° cent.

GIVEN ITEMS.

$W = 1500$ watts; $e = 110$ volts; $V_0 = 2$ per cent;
$l = 250$ feet; $K = 0.85$ lag.

From (98), $$v = \frac{0.02 \times 110}{1 - 0.02} = 2.25 \text{ volts.}$$

From Table 37, page 82, $a = 963 \times 0.98 = 944$.

From (94) (in which B from Table 38, page 82, $= 0.500$),

$$I = \frac{0.5 \times 1500}{110 \times 0.85} = 8.0 \text{ amperes.}$$

REQUIRED ITEMS.

Size of each wire. — From (84),

$$M = \frac{944 \times 2.25}{8 \times 250} = 1.06.$$

From Table 40, page 84, use No. 8, for which $M = 1.18$.

Per cent volt loss. — From (86),

$$V = \frac{1.18 \times 8 \times 250}{0.01 \times 944 \times 110} = 2.27 \text{ per cent.}$$

Volts at source. — From (88), $e_0 = 110\ (1 + 0.0227) = 113$ volts.

Example 30. — A 125 volt, 125 cycle generator is to supply a three-phase load of 100 amperes per wire at 100 per cent power-factor over 225 feet of cable, with a loss of 3 per cent of the generated volts. The wires are to have a conductivity of 99 per cent and a temperature of 30° cent.

GIVEN ITEMS.

$I = 100$ amperes; $e_0 = 125$ volts; $V_0 = 3$ per cent;
$l = 225$ feet; $K = 1.00$.

From (98), $v = 0.03 \times 125 = 3.75$ volts.
From Table 37, page 82,

$$a = 1110 \times 0.99 = 1100; \quad b = \frac{179}{0.99} = 181.$$

REQUIRED ITEMS.

Size of each wire. — From (84),

$$M = \frac{1100 \times 3.75}{100 \times 225} = 0.183.$$

From Table 39, page 77, use No. 00, for which $M = 0.163$.

Volt loss. — From (85),

$$v = \frac{0.163 \times 100 \times 225}{1100} = 3.3 \text{ volts.}$$

Per cent volt loss. — From (98),

$$V_0 = \frac{3.3}{0.01 \times 125} = 2.6 \text{ per cent.}$$

Volts at load. — From (89), $e = 125 - 3.3 = 121.7$ volts.

Formulas for A.C. Interior Wiring.

Required Items.	For Voltage Given at Load.	For Voltage Given at Source.
Size of each wire.	a from Table 37, page 82.	
	$M = \frac{av}{Il}$ . . . . . (84)	
	Then find wire from Table 39 or 40, page 83 or 84.	
Volt loss.	$v = \frac{MIl}{a}$ . . . . . (85)	
Per cent volt loss.	$V = \frac{MIl}{0.01\,ae} = \frac{v}{0.01\,e}$. (86)	$V_0''' = \frac{Mwl}{0.01\,ae_0^2}$ . . . (87)
		V_0 from Table 31, page 72.
Volts at source or load.	$e_0 = e + v$ $= e(1 + 0.01V)$. (88)	$e = e_0 - v$ $= e_0(1 - 0.01V_0)$. (89)
Per cent power loss.	r from Table 3, page 7 – b from Table 37, page 82.	
	$P = \frac{rbIl}{eK}$ . . . . . (90)	$P = \frac{rbIl}{e_0(1-0.01V_0)K}$. (91)
Watt loss.	$p = 0.01\,Pw$ . . . . . (92)	
Watts at source.	$w_0 = w + p = w(1 + 0.01\,P)$ . . . . (93)	
Amperes per wire. (See Par. 34, page 35.)	B from Table 38, page 82.	
	$I = \frac{Bw}{eK} = \frac{Bw'}{e}$ . . (94)	$I = \frac{Bw}{e_0(1-0.01V_0)K}$ $= \frac{Bw'}{e_0(1-0.01V_0)}$ . . (95)
Power-factor at source.	$K_0 = \frac{(1+0.01\,P)K}{1+0.01V} = (1+0.01P)(1-0.01V_0)K$. (96)	

When given V, find v from $v = 0.01Ve = 0.01Ve_0/(1 + 0.01V)$ . . . (97)
When given V_0, find v from $v = 0.01V_0e_0 = 0.01V_0e/(1 - 0.01V_0)$ . . (98)
When given w or w', find I from (94) or (95) above.

NOTATION.

a = Transmission factor. Table 37, page 82.
B = Transmission constant. Table 38, p. 82.
b = Transmission factor. Table 37, p. 82.
e = Volts between wires at load. See par. 32, p. 35.
e_0 = Volts between wires at source. See par. 32, p. 35.
I = Amperes per wire. See par. 34, p. 35.
K = Power-factor of load, in decimals.
K_0 = Power-factor at source, in decimals.
l = Distance from source to load, in feet.
M = Wire factor. Tables 39 and 40, pp. 83 and 84.
P = Power loss in per cent of power at load.
p = Total power loss in watts.
r = Resistance per foot of one wire. Table 3, p. 7.
V = Volt loss in per cent of volts at load.
V_0 = Volt loss in per cent of volts at source.
V_0''' = Volt loss factor. Table 31, p. 72.
v = Volt loss (volts at source – volts at load).
w = Real watts at load.
w_0 = Real watts at source.
w' = Volt-amperes (apparent power) at load.

Table 37. — Values of *a* and *b* for Balanced Loads.

Temperature in Degrees.		1-Phase System with 2 Equal Wires.		2-Phase System with 3 Equal Wires.		2-Phase System with 4 Equal Wires.		3-Phase System with 3 Equal Wires.	
Cent.	Fahr.	*a*	*b*	*a*	*b*	*a*	*b*	*a*	*b*
0	32	1080	185	895	185	1080	185	1250	160
10	50	1040	193	860	193	1040	193	1200	167
20	68	1000	200	828	200	1000	200	1160	173
30	86	963	208	798	208	963	208	1110	179
40	104	928	215	769	215	928	215	1070	186
50	122	896	223	742	223	896	223	1030	193

The above values are for copper of 100 per cent conductivity. For other conductivity, multiply *a* and divide *b* by given conductivity.

Table 38. — Values of *B* for Balanced Loads.

1-Phase System.	2-Phase System.	3-Phase System.
1.000	0.500	0.578

Table 39. — Values of *M* for Copper Wires in Interior Conduits.

B. & S. or A.W.G.	25 Cycles per Second.							40 Cycles per Second.							60 Cycles per Second.							125 Cycles per Second.						
	Lagging Power-Factor of Load in Per Cent.																											
	100	98	95	90	85	80	75	100	98	95	90	85	80	75	100	98	95	90	85	80	75	100	98	95	90	85	80	75
0000	.098	.100	.101	.099	.097	.095	.092	.099	.104	.106	.106	.105	.101	.102	.100	.110	.113	.116	.117	.116	.115	....	.128	.139	.151	.158	.161	.165
000	.124	.126	.126	.123	.120	.117	.112	.124	.129	.131	.130	.128	.136	.122	.125	.134	.137	.138	.139	.137	.135	.131	.152	.162	.173	.179	.182	.184
00	.155	.157	.157	.152	.148	.144	.138	.157	.161	.162	.160	.157	.153	.148	.157	.166	.169	.169	.168	.165	.162	.163	.184	.192	.205	.207	.209	.210
0	.196	.198	.196	.190	.183	.177	.171	.197	.201	.201	.197	.192	.187	.181	.196	.205	.207	.206	.203	.199	.195	.202	.223	.231	.240	.242	.242	.242
1	.247	.249	.245	.237	.228	.220	.211	.248	.252	.250	.244	.236	.230	.222	.249	.256	.257	.254	.248	.243	.236	.252	.274	.281	.288	.289	.286	.284
2	.312	.312	.307	.296	.284	.274	.262	.312	.316	.312	.303	.293	.284	.272	.312	.319	.318	.313	.306	.299	.289	.316	.338	.344	.348	.347	.342	.338
3	.394	.394	.385	.372	.354	.342	.328	.394	.398	.392	.378	.367	.353	.337	.394	.401	.397	.389	.377	.368	.352	.398	.420	.425	.426	.422	.415	.406
4	.496	.495	.483	.464	.444	.430	.409	.496	.500	.490	.471	.457	.441	.421	.496	.504	.497	.484	.470	.456	.438	.497	.520	.525	.522	.515	.506	.491
5	.625	.622	.607	.583	.557	.540	.511	.625	.627	.615	.590	.571	.551	.525	625	.632	.623	.604	.582	.564	.540	.625	.650	.652	.645	.631	.618	.599
6	.788	.784	.763	.733	.698	.673	.639	.788	.789	.770	.740	.710	.686	.654	.788	.792	.778	.753	.726	.703	.670	.788	.814	.810	.794	.776	.758	.730
8	1.25	1.24	1.21	1.15	1.10	1.05	1.00	1.25	1.24	1.21	1.16	1.12	1.07	1.02	1.25	1.24	1.23	1.18	1.14	1.09	1.04	1.25	1.27	1.26	1.22	1.18	1.15	1.09
10	1.99	1.97	1.91	1.83	1.74	1.67	1.58	1.99	1.98	1.92	1.84	1.76	1.68	1.60	1.99	1.98	1.93	1.86	1.77	1.71	1.61	1.99	2.01	1.97	1.90	1.83	1.77	1.69
12	3.18	3.13	3.04	2.91	2.76	2.65	2.51	3.18	3.14	3.04	2.92	2.78	2.67	2.53	3.18	3.14	3.06	2.93	2.80	2.68	2.55	3.18	3.16	3.09	2.98	2.86	2.77	2.62
14	5.04	4.98	4.82	4.60	4.38	4.21	3.99	5.04	4.98	4.82	4.62	4.40	4.21	4.00	5.04	4.98	4.85	4.64	4.42	4.24	4.00	5.04	5.01	4.89	4.68	4.49	4.32	4.08
16	8.02	7.91	7.64	7.30	6.95	6.68	6.30	8.02	7.92	7.64	7.30	6.97	6.68	6.32	8.02	7.93	7.69	7.34	6.99	6.69	6.35	8.02	7.95	7.72	7.40	7.06	6.74	6.43
18	12.7	12.6	12.1	11.6	11.0	10.6	10.0	12.7	12.6	12.1	11.6	11.1	10.6	10.0	12.7	12.6	12.2	11.6	11.1	10.6	10.1	12.7	12.6	12.2	11.6	11.1	10.6	10.1

Table 40. — Values of *M* for Copper Wires in Molding or on Cleats.

B. & S. or A.W.G.	25 Cycles per Second.							40 Cycles per Second.							60 Cycles per Second.							125 Cycles per Second.						
	Lagging Power Factor of Load in Per Cent.																											
	100	98	95	90	85	80	75	100	98	95	90	85	80	75	100	98	95	90	85	80	75	100	98	95	90	85	80	75
0000	.099	.107	.110	.111	.111	.111	.109	.102	.115	.120	.125	.128	.129	.130	....	.127	.136	.148	.152	.156	.158	....	....	....	.214	.230	.242	.255
000	.125	.132	.134	.134	.133	.132	.129	.127	.140	.145	.149	.152	.152	.151	.131	.152	.160	.170	.175	.179	.180	....	....	.208	.237	.254	.267	.279
00	.156	.163	.164	.164	.162	.160	.155	.159	.172	.176	.180	.181	.180	.178	.163	.184	.192	.201	.206	.208	.209	....	....	.248	.270	.288	.301	.306
0	.197	.204	.204	.203	.199	.195	.189	.198	.212	.216	.218	.218	.215	.211	.202	.224	.233	.239	.243	.244	.243	....	.267	.289	.317	.329	.340	.347
1	.248	.255	.254	.252	.245	.238	.229	.250	.263	.266	.265	.263	.260	.253	.256	.275	.283	.288	.288	.288	.285	....	.318	.340	.366	.378	.388	.392
2	.312	.319	.315	.310	.300	.292	.280	.313	.326	.327	.325	.320	.314	.304	.316	.337	.344	.347	.347	.342	.334	.330	.381	.401	.424	.436	.443	.447
3	.394	.399	.394	.385	.371	.361	.346	.395	.406	.406	.400	.392	.383	.368	.398	.419	.423	.423	.418	.413	.402	.409	.461	.482	.501	.511	.516	.517
4	.496	.500	.492	.477	.460	.447	.427	.496	.508	.503	.496	.482	.470	.450	.498	.518	.521	.518	.510	.501	.485	.508	.561	.579	.596	.603	.605	.600
5	.626	.629	.614	.594	.572	.555	.529	.626	.636	.628	.614	.593	.579	.554	.627	.646	.644	.636	.622	.609	.588	.636	.689	.705	.715	.719	.717	.707
6	.788	.788	.770	.742	.715	.691	.657	.788	.795	.783	.760	.734	.714	.683	.789	.806	.799	.782	.766	.747	.718	.796	.843	.858	.867	.862	.852	.836
8	1.25	1.24	1.22	1.16	1.12	1.07	1.02	1.25	1.25	1.23	1.18	1.14	1.10	1.05	1.25	1.26	1.25	1.20	1.18	1.13	1.19	1.26	1.30	1.30	1.29	1.27	1.25	1.20
10	1.99	1.98	1.92	1.84	1.76	1.69	1.60	1.99	1.99	1.94	1.86	1.78	1.71	1.63	1.99	2.00	1.95	1.88	1.81	1.75	1.66	1.99	2.04	2.01	1.97	1.92	1.87	1.80
12	3.18	3.14	3.05	2.92	2.78	2.67	2.53	3.18	3.15	3.06	2.94	2.80	2.70	2.56	3.18	3.16	3.08	2.96	2.84	2.73	2.70	3.18	3.19	3.12	3.05	2.95	2.87	2.73
14	5.04	4.99	4.82	4.60	4.39	4.23	3.99	5.04	4.99	4.83	4.64	4.42	4.25	4.02	5.04	5.00	4.86	4.67	4.46	4.29	4.05	5.04	5.03	4.91	4.75	4.57	4.42	4.19
16	8.02	7.93	7.67	7.30	6.97	6.70	6.31	8.02	7.94	7.69	7.32	7.00	6.72	6.34	8.02	7.95	7.70	7.37	7.03	6.74	6.40	8.02	7.97	7.74	7.47	7.15	6.83	6.54
18	12.7	12.6	12.1	11.6	11.0	10.6	10.0	12.7	12.6	12.1	11.6	11.1	10.6	10.0	12.7	12.6	12.2	11.6	11.1	10.7	10.1	12.7	12.6	12.2	11.7	11.2	10.8	10.2

CHAPTER VI.

DISTRIBUTION FOR SINGLE-PHASE RAILWAYS.

57. **Introduction.** — A trolley circuit consists primarily of an overhead trolley wire suspended from single, double or multiple catenary construction, and the track rails. The trolley wire may be in parallel with an auxiliary feeder or " by-pass." However, the installation of auxiliary feeders is the exception rather than the rule, and since their position with respect to the trolley wire and rails is of much importance but of wide variation, no attempt has been made to include them in the results. The table of M has been calculated for 15 and 25-cycle systems with one and two bonded rails per track. The rails have a relative resistance of steel to copper of 12, while the trolley wires are taken at 100 per cent conductivity and 20° cent.

58. **Method of Calculation.** — The value of M is based on the " Report of Electric Railway Test Commission " and on isolated calculations published in various places. General laws have been deduced for the grounded portion of the circuit, and they have been found to agree sufficiently well with the experimental results. The properties of the trolley and catenary construction have been calculated and the results have been combined with those for the rails to obtain the values of the complete circuit. The formulas on page 92 are similar to those for overhead transmission lines except that the former have been expressed in terms of amperes in order to facilitate computation. The per cent volt loss is also given in terms of the kilowatts at load in order to avoid approximation when the voltage is known only at the source.

59. **Impedance of Rail.**— Due to skin effect, an alternating current flowing through a steel rail has a diminishing density toward the center, thereby increasing the effective resistance of the rail. Eddy currents and hysteresis losses produce a further increase in the effective resistance, resulting in an impedance to alternating current considerably greater than the resistance of the

rail to direct current. The ratio of impedance divided by the resistance to direct current is called the impedance ratio.

The impedance of a complete track depends upon the frequency of the system, on the height of the trolley wire, on the shape, size, permeability and specific resistance of the rail, and on the character of bonds and roadbed. It is evidently a complex quantity and can be determined only by tests of installed track under normal conditions of operation.

60. Permeability of Rail. — The permeability of a given rail depends upon the density of current. The method of variation is irregular but an average value of the permeability can be assumed without causing much error in the final result.

61. Impedance and Weight of Rail. — The impedance depends on the size and shape of rail and differs in the standard bullhead section of foreign practice from the T-rail of this country. It changes very slightly with the weight of the rail commonly installed. In any case the impedance has been found by experiment to vary inversely with the perimeter of the section.

62. Impedance of Rail and Frequency. — Experiment shows, at least at lower frequencies, that the impedance of rails varies directly as the square root of the cycles per second. Thus at 25 cycles per second the impedance is about 1.3 times the corresponding value at a frequency of 15.

63. Formula for Rail Impedance. — It has been concluded from experiments that the impedance of rails varies directly as the square root of the frequency and inversely as the perimeter. However, the perimeter of standard T-rails is approximately proportional to the square root of their weight per yard. The author has developed the following formula and has found that it agrees sufficiently well with tests to serve for purposes of practical calculation.

Impedance per mile of one T-rail, bonded and installed,

$$= 0.8\sqrt{\frac{\text{Cycles per second}}{\text{Pounds per yard}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

The constant 0.8 is based on an impedance ratio of 6.6 at 25 cycles per second for a 75-pound bonded rail of a relative resistance of steel to copper of about 12, with the trolley wire about 20 feet above the track.*

* General Electric Co.'s Bulletin, No. 4392, 1904.

64. Power-Factor of Track. — The power-factor increases with the current, but at the lower frequencies this variation is small for normal current densities. Tests of the Electric Railway Commission show a slight increase of power-factor with the weight of rail but other experiments indicate practically no change.* The power-factor of the track has been assumed constant for all rail weights, at 65 per cent for 15 cycles and 55 per cent for 25 cycles per second.

65. Height of Trolley. — Trolley wires for single-phase systems range in height from about 18 to 22 feet above rails. The height assumed in the calculations of M is 22 feet, but any variation within standard limits will have small effect on the impedance of the trolley, and almost no effect on the impedance of the track.

66. Effect of Catenary Construction. — The trolley wire is supported from a single catenary wire, or from two wires arranged either horizontally or vertically. In almost every case the trolley and catenary wires are electrically connected. However, since the catenary wires are steel, their skin effect prevents them from carrying more than a small part of the current, which for single catenary construction has been determined by calculation to be equivalent to about 10 per cent.† In double or multiple catenary construction the proportional part of the current carried by the trolley is somewhat reduced, but in order to be safe in calculation for either type of overhead construction, the above result has been used.

67. Impedance of Complete Circuit. — The ohmic resistance and the reactance of the track and trolley having been calculated, the two were combined to obtain the total impedance. Table 42, page 93, gives the values of M. It will be observed that the effect of varying the rail weight is less pronounced than of varying the size of trolley, or, in other words, that the greater part of the total impedance is due to the trolley. Since the impedance of the overhead system is susceptible of fairly accurate calculation, it follows that any error and variation in the assumed value of the track impedance will have a small effect on the final result. This conclusion is corroborated by tests.

Following is a comparison between the calculated impedance

* Foster's "Electrical Engineer's Pocketbook," 1908, page 795.

† J. B. Whitehead, Proceedings of A.I.E.E., May, 1908, page 627.

per mile and the published results of tests on installed lines. The values of the Electric Railway Test Commission are not included, as their test track is not similar to modern construction.

Table 41. — Test and Calculated Values of Impedance Per Mile at 25 Cycles Per Second.

Rails.	Trolleys.	Test Values.	Calculated Values.
4-75 lb.	2 No. 000	0.31 (a)	0.34
2-70 lb.	1 No. 000	0.60 (b)	0.68
2-80 lb.	1 No. 000	0.65 to 0.70 (c)	0.67

(a) General Electric Co.'s Tests on Ballston Line, Bulletin 4392, 1904.

(b) Based on Westinghouse Tests, Foster's "Electrical Engineer's Pocketbook," 1908 page 797.

(c) J. B. Whitehead, Proceedings of A. I. E. E., May, 1908, page 638; based on tests and calculations.

68. Multiple Tracks. — With small error the results may be applied to a number of tracks, either by proportioning the total load or by dividing the wire factor in Table 42 for one track by the number of tracks. Thus the value of M at 25 cycles per second and 80 per cent load power-factor for four 80-pound rails and two No. 000 trolleys is

$$\frac{0.065}{2} = 0.0325.$$

69. Examples. — For convenience in computation the formulas have been expressed in terms of current. The per cent volt loss has also been given in terms of kilowatts at the load, so that no preliminary approximation is required when the voltage is known only at the source. The meaning of the term "source" is stated in paragraph 32, page 35.

The application of the formulas on page 92 is illustrated in the following typical examples.

Example 31. — Calculate the volt loss per mile per 100 amperes at a load power-factor of 80 per cent for 15 and 25-cycle single-phase railway circuits as follows:

Two 70-lb. rails and one No. 00 trolley.
Four 80-lb. rails and two No. 000 trolleys.
Eight 100-lb. rails and four No. 0000 trolleys.

From (107) and Table 42, page 93, the following results are obtained:

Rails.	Trolleys.	Volt Loss per Mile per 100 Amperes.	
		15 Cycles per Sec.	25 Cycles per Sec.
2-70 lb.	1 No. 00	61.0	73.0
4-80 lb.	2 No. 000	26.0	32.5
8-100 lb.	4 No. 0000	11.5	14.5

Example 32. — A 25-cycle single-phase railway system having two 70-lb. bonded rails is to be designed for a loss of 10 per cent of the 3300 generated volts when delivering 150 amperes at 90 per cent power-factor to a car 4 miles away.

GIVEN ITEMS.

$I = 150$ amperes; $E_0 = 3.3$ kv.; $V_0 = 10$ per cent;
$L = 4$ miles; $K = 0.90$ lag.

REQUIRED ITEMS.

Size of trolley and rail. — From (101),

$$M = \frac{3.3 \times 10}{150 \times 4} = 0.055.$$

From Table 42, page 93, use No. 0000 trolley wire, for which $M = 0.057$.

Per cent volt loss. — From (103),

$$V_0 = \frac{0.057 \times 150 \times 4}{3.3} = 10.4 \text{ per cent.}$$

Kilovolts at car. — From (105), $E = 3.3\,(1 - 0.104) = 2.96$ kv.
Per cent power loss.—From Table 42, page 93, $R = 0.364$ ohm.
From (109),

$$P = \frac{0.364 \times 150 \times 4}{10 \times 3.3\,(1 - 0.104)\,0.90} = 8.2 \text{ per cent.}$$

Power-factor at source. — From (114),

$$K_0 = (1 + 0.082)\,(1 - 0.104)\,0.90 = 0.87 \text{ lag.}$$

Example 33. — A 25-cycle single-phase road with four 80-lb. bonded rails and two No. 000 trolleys supplies each of three cars with 50 amperes at 85 per cent power-factor when located at 3, 5, and 10 miles away, respectively, the power station voltage being 11,000.

Owing to the power-factor at the first two cars, the following solution is slightly in error.

GIVEN ITEMS.

$$IL = 50\ (3 + 5 + 10) = 900 \text{ ampere-miles.}$$

From Table 42, page 93, $M = \frac{0.063}{2} = 0.0315.$

REQUIRED ITEMS.

Per cent volt loss to last car. — From (103),

$$V_0 = \frac{0.0315 \times 900}{11} = 2.6 \text{ per cent.}$$

Kilovolts at last car. — From (106), $E = 11\ (1 - 0.026) = 10.7$ kv.

Example 34. — A 25-cycle single-phase car starting 8 miles from a transformer station generating 6600 volts takes 500 kw. at 80 per cent power-factor from a circuit consisting of two 70-lb. rails and one No. 000 trolley.

GIVEN ITEMS.

$W = 500$ kw.; $E_0 = 6.6$ kv.; $L = 8$ miles; $K = 0.80$ lag.

From Table 42, page 93, $M = 0.066.$

REQUIRED ITEMS.

Per cent volt loss. — From (104),

$$V_0''' = \frac{0.066 \times 500 \times 8}{(6.6)^2\ 0.80} = 7.6.$$

From Table 31, page 72, $V_0 = 8.3$ per cent.

Kilovolts at load. — From (106), $E = 6.6\ (1 - 0.083) = 6.05$ kv.

Amperes. — From (112),

$$I = \frac{500}{6.05 \times 0.80} = 103 \text{ amperes.}$$

Per cent power loss. — From Table 42, page 93, $R = 0.425$ ohm.

From (108), $P = \frac{0.425 \times 103 \times 8}{10 \times 6.05 \times 0.80} = 7.2$ per cent.

Kilowatts at source. — From (111),

$$W_0 = 500\ (1 + 0.072) = 536 \text{ kw.}$$

Power-factor at source. — From (114),

$$K_0 = (1 + 0.072)\ (1 - 0.083)\ 0.80 = 0.79 \text{ lag.}$$

Example 35. — Two 15-cycle single-phase locomotives start simultaneously 10 miles from a power station generating 11,000 volts. Determine the line voltage at the locomotives if each takes 3000 kv.-amp. (kilovolt-amperes) at 75 per cent power-factor, over a circuit consisting of eight 100-lb. rails and four No. 0000 trolley.

GIVEN ITEMS.

$W' = 6000$ kv.-amp.; $E_0 = 11$ kv.; $L = 10$ miles; $K = 0.75$ lag.

From Table 42, page 93,

$$M = \frac{0.046}{4} = 0.0115.$$

$$W = 6000 \times 0.75 = 4500 \text{ kw.}$$

REQUIRED ITEMS.

Per cent volt loss. — From (104),

$$V_0''' = \frac{0.0115 \times 4500 \times 10}{(11)^2\ 0.75} = 5.7.$$

From Table 31, page 72, $V_0 = 6.1$ per cent.

Kilovolts at locomotives. — From (106),

$$E = 11\ (1 - 0.061) = 10.3 \text{ kv.}$$

Formulas for Single-Phase Railway Circuits.

Required Items.	For Voltage Given at Load.	For Voltage Given at Source.
Size of trolley and rail.	$M = \frac{EV}{IL}$ . . . . . (100)	$M = \frac{E_0 V_0}{IL}$ . . . (101)
Per cent volt loss.	$V = \frac{MIL}{E}$ . . . . . (102)	$V_0 = \frac{MIL}{E_0}$ . . . (103)
		$V_0''' = \frac{MWL}{E_0^2 K}$ . . . (104)
		V_0 from Table 31, page 72.
Kilovolts at source or load.	$E_0 = E(1 + 0.01V)$. (105)	$E = E_0(1 - 0.01V_0)$. (106)
Volt loss.	$v = 1000(E_0 - E) = 10VE = 10V_0E_0 = 10MIL$. (107)	
Per cent power loss.	R from Table 42, page 93.	
	$P = \frac{RIL}{10EK}$ . . . . (108)	$P = \frac{RIL}{10E_0(1 - 0.01V_0)K}$ (109
Watt loss.	$p = 10PW = RI^2L$ . . . . . (110)	
Kilowatts at source.	$W_0 = W + 0.001p = W(1 + 0.01P)$ . . . . . (111)	
Amperes.	$I = \frac{W}{EK} = \frac{W'}{E}$ . . (112)	$I = \frac{W}{E_0(1 - 0.01V_0)K} = \frac{W'}{E_0(1 - 0.01V_0)}$. (113)
Power-factor at source.	$K_0 = \frac{(1 + 0.01P)K}{1 + 0.01V} = (1 + 0.01P)(1 - 0.01V_0)K$. (114)	

When given E and V_0, find V from $V = V_0/(1 - .01V_0)$ . . . . . (115)
When given E_0 and V, find V_0 from $V_0 = V/(1 + .01V)$ . . . . . (116)
When given W or W', find I from (112) or (113) above.

NOTATION.

E = Kilovolts between trolley and rail, at load.
E_0 = Kilovolts between trolley and rail, at source.
I = Total amperes from source.
K = Power-factor of load, in decimals.
K_0 = Power-factor at source, in decimals.
L = Distance from source to load, in miles.
M = Wire factor. Table 42, page 93.
P = Power loss in per cent of power at load.
p = Power loss in watts.
R = Resistance per mile of circuit. Table 42, page 93.
V = Volt loss in per cent of volts at load.
V_0 = Volt loss in per cent of volts at source.
V_0''' = Volt loss factor. Table 31, page 72.
v = Total loss in volts (volts at source – volts at load).
W = Real kilowatts at load.
W_0 = Real kilowatts at source.
W' = Kilovolt-amperes (apparent power) at load.

Table 42. — Values of *M* for Single-Phase Railway Circuits.

Rail. Lb. per Yard	Trolley. Cir. Mils. or A.W.G.	15 Cycles per Second. Ohms per Mile of Circuit		1 Rail and 1 Trolley. Lagging Power-Factor of Load in Per Cent.					2 Rails and 1 Trolley. Lagging Power-Factor of Load in Per Cent.					25 Cycles per Second. Ohms per Mile of Circuit.		1 Rail and 1 Trolley. Lagging Power-Factor of Load in Per Cent.					2 Rails and 1 Trolley. Lagging Power-Factor of Load in Per Cent.				
		1 Rail.	2 Rails.	98	90	80	70	60	98	90	80	70	60	1 Rail.	2 Rails.	98	90	80	70	60	98	90	80	70	60
100	250000	.398	.298	.049	.055	.058	.058	.058	.037	.041	.043	.043	.043	.417	.307	.057	.068	.073	.076	.078	.042	.050	.054	.057	.058
	0000	.433	.333	.052	.058	.061	.061	.061	.040	.044	.046	.046	.045	.452	.342	.061	.072	.077	.079	.080	.046	.054	.058	.060	.061
	000	.494	.394	.058	.064	.066	.066	.065	.046	.050	.051	.051	.050	.513	.403	.067	.077	.082	.084	.085	.052	.059	.062	.064	.065
	00	.571	.471	.066	.071	.072	.071	.070	.053	.057	.057	.056	.055	.590	.480	.074	.084	.088	.089	.090	.059	.066	.069	.070	.070
90	250000	.409	.303	.050	.057	.059	.060	.060	.037	.042	.043	.044	.044	.429	.313	.059	.070	.076	.078	.080	.043	.052	.055	.058	.059
	0000	.444	.338	.054	.060	.062	.063	.063	.041	.045	.046	.047	.046	.464	.348	.063	.074	.079	.081	.083	.047	.056	.059	.061	.062
	000	.505	.399	.060	.065	.067	.068	.067	.046	.050	.051	.051	.050	.525	.409	.068	.079	.084	.085	.087	.053	.060	.064	.065	.066
	00	.582	.476	.067	.072	.074	.074	.073	.054	.057	.058	.057	.056	.602	.486	.076	.086	.090	.091	.092	.060	.067	.070	.071	.071
80	250000	.422	.310	.052	.058	.061	.062	.062	.038	.043	.044	.045	.045	.444	.321	.061	.073	.078	.081	.083	.044	.053	.057	.059	.061
	0000	.457	.345	.056	.062	.064	.065	.064	.041	.046	.047	.048	.047	.479	.356	.065	.076	.081	.084	.085	.048	.056	.060	.062	.063
	000	.518	.406	.061	.067	.069	.069	.068	.047	.051	.052	.052	.051	.540	.417	.070	.081	.086	.088	.089	.054	.061	.065	.066	.067
	00	.595	.483	.069	.074	.076	.075	.074	.054	.058	.059	.058	.056	.617	.494	.078	.088	.093	.094	.095	.061	.068	.072	.072	.072
70	250000	.438	.318	.054	.061	.063	.064	.064	.039	.044	.046	.046	.046	.461	.329	.063	.075	.081	.084	.086	.046	.054	.058	.061	.062
	0000	.473	.353	.058	.064	.066	.067	.067	.042	.047	.049	.049	.048	.496	.364	.067	.079	.084	.087	.088	.049	.057	.061	.063	.064
	000	.534	.414	.063	.069	.071	.072	.071	.048	.052	.054	.053	.052	.557	.425	.072	.084	.089	.091	.092	.055	.063	.066	.068	.069
	00	.611	.491	.071	.076	.078	.078	.077	.056	.060	.061	.060	.058	.634	.502	.080	.091	.096	.097	.098	.062	.070	.073	.074	.074
60	250000	.457	.327	.057	.063	.066	.067	.067	.040	.045	.047	.047	.047	.481	.339	.066	.079	.085	.088	.090	.046	.056	.060	.063	.064
	0000	.492	.362	.061	.067	.070	.070	.070	.044	.049	.050	.050	.050	.516	.374	.069	.082	.087	.090	.092	.050	.059	.063	.065	.066
	000	.553	.423	.066	.072	.075	.075	.074	.049	.054	.055	.054	.054	.577	.435	.075	.087	.092	.095	.096	.056	.065	.069	.070	.071
	00	.630	.500	.073	.079	.081	.080	.079	.057	.061	.061	.060	.059	.654	.512	.083	.094	.099	.101	.102	.063	.071	.075	.075	.075

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